Essays in Macroeconomics

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Dedication

To my parents, Cecilia and Sergio, whose love and example has been my drive for the last 36 years

Abstract

This dissertation consists of three chapters. In the first essay of my thesis, I document a marked declined in the share of entrepreneurial households in the United States and I propose and quantify a mechanism to account for such decline. Using individual-level data, I provide evidence on the decline in the population share of entrepreneurs and in the entry rate into entrepreneurship. I also show that the decline is most concentrated among college graduates. Then, using an otherwise standard entrepreneurial choice model with two skill groups of individuals, I show that the decline in entrepreneurship is the equilibrium outcome of two forces that have increased the returns to high skill labor: the skill-biased technical change and the decrease in the cost of capital goods. I find that these two technological forces jointly account for three-quarters of the decline in the share of entrepreneurs observed in the United States over the last 30 years. In the second essay of this thesis, Nicholas Bloom, Fatih Guvenen, and I study the cyclicality the distribution of firm-leve outcomes over the business cycle. Using firm-level panel data from the US Census and for more than forty other countries, we show that the skewness of the growth rate of employment and sales is procyclical. In particular, during recessions, they display a large left tail of negative growth rates (and during booms, a large right tail of positive growth rates). These results are robust to different selection criteria, across countries, industries, and measures. We find similar results at the industry level: industries with falling growth rates see more left-skewed growth rates of firm sales. We then build a heterogeneous agents model in which entrepreneurs face shocks with time-varying skewness that matches the firm-level distributions we document for the United States. Our quantitative results show that a negative shock to the skewness of firms' productivity growth (keeping the mean and variance constant) generates a significant and persistent drop in output, investment, hiring, and consumption. In the third and last essay of this thesis, Mons Chan, Min Xu, and I, study the importance of fluctuation of firm-level productivity in explaining the fluctuations in workers' wages. In particular, we use matched employer-employee data from Denmark to analyze the extent to which firms' productivity shocks are passed to workers' wages. The richness of our dataset allows us to separately study continuing and non-continuing

workers, to correct for selection, and to investigate how the passthrough varies across narrow population groups. Our results show a much larger degree of passthrough from firms' shocks to workers' wages than reported in previous research. On average, an increase of one standard deviation in firm-level TFP commands an increase of 3.0% in annual wages ($1,500 USD for the average worker). Furthermore, we find that the effect of productivity shocks on wage growth for workers who switch firms is larger relative to workers that stay within the same firm. Finally, we find large differences in the passthrough for workers of different income levels, ages, industries, and firms of different productivity levels. In the second part of our paper, we estimate a stochastic process of income that captures the salient features the data. We then embed the estimated stochastic process into a life-cycle consumption savings model to evaluate the welfare and distributional implications of the passthrough from firms' TFP shocks to workers' wages.

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Chapter 1

Technical Change and Entrepreneurship

1.1 Introduction

Entrepreneurs are widely considered the backbone of the US economy. However, an increasing number of studies document a significant decline in the pace of formation of new businesses and other measures of entrepreneurship starting in the early 1980s.[1][2] This decrease in entrepreneurship is at the center of the decline in dynamism experienced by the US economy in recent decades (Davis and Haltiwanger, 2014). This has raised concern among scholars and policymakers because of the importance of entrepreneurs for productivity and economic growth.[3]

[1] I am extremely grateful to my advisors, Fatih Guvenen, Patrick J. Kehoe, and Elena Pastorino for many helpful comments and encouragement. I also thank Nicholas Bloom, Simone Civale, Luis Díez-Catalán, Carlos Garriga, Eugenia González-Aguado, Parisa Kamali, Sergio Ocampo, John Shea, Guillaume Sublet, Tasaneeya Viratyosin, and seminar participants at the Macro-Labor Workshop at the University of Minnesota, the Innovation and Inequality Workshop at the University of Washington in St. Louis (October 2017), Minneapolis Fed, Dallas Fed, University of Maryland, UC San Diego, UCLA, Georgetown, and Wharton for helpful comments and discussions.

[2] Several other papers have discussed the decline in the pace of creation of new businesses and entrepreneurship. Reedy and Strom (2012); Decker, Haltiwanger, Jarmin, and Miranda (2014, 2015) show evidence on the decline in the startup rate (the share of the firm population accounted for by age-zero firms) and in the share of fast-growing firms (which are disproportionally young). Pugsley and Sahin (2014) show the evidence of increasing concentration of economic activity on older and larger firms.

[3] See, for instance Haltiwanger, Decker, and Jarmin (2015) and Yellen (2016).

Previous researchers have proposed that the decline in firm creation maybe be driven by an increase in the cost to start a firm, stemming possibly from an increase in regulation (Davis and Haltiwanger, 2014), or a shift towards an older population (Karahan, Pugsley, and Sahin, 2016). However, in this paper I propose that the decline in entrepreneurship is the equilibrium response to technological improvements that have changed the incentives of individuals to start their own business. In particular, I show that the same aggregate forces that have resulted in an increase in the returns to high skill labor, namely, the skill-biased technical change (Krueger, 1993) and the decrease in the cost of capital goods (Krusell, Ohanian, Ríos-Rull, and Violante, 2000), account for a significant fraction of the decrease in entrepreneurship observed in the United States since the mid-1980s.[4]

The first contribution of this paper is to provide new evidence on the decline of entrepreneurship experienced by the US economy over the last three decades. Using individual-level data from the Panel Study of Income Dynamics (PSID) I show that the decline in the pace of firm creation has been accompanied by a decline in the share of entrepreneurs in the US working-age population. Specifically, I show that the population share of entrepreneurs declined from 7.8% in 1985 to 3.9% in 2014. Moreover, the share of individuals transitioning into entrepreneurship declined by a half over the same period. By separating the population into different education groups, I find that the decline in entrepreneurship is most concentrated among the college graduates. In fact, the share of college graduates who are entrepreneurs declined from 12.2% in 1985 to 5.3% in 2014, whereas the share of non-college graduate entrepreneurs declined from 4.7% to 2.7% over the same period.[5] Finally, I provide evidence of an increase in selection into entrepreneurship. Using past labor earnings as a measure of individual skill, I show that among college graduates, the average past labor income of new entrepreneurs increased by 35 log points over the last 30 years, whereas the average past labor earnings among individuals that stayed as workers

[4]The rapid increase of the returns to high skill workers has been extensively documented. See, for instance, Acemoglu (2002), Autor, Katz, and Kearney (2008), or Acemoglu and Autor (2011) and the references therein.

[5]Here I consider a sample of heads of household from the PSID between 22 and 60 years of age. I classify as entrepreneurs those heads of household in the PSID for whom four conditions hold: (i) the household owns a business, (ii) the head is self-employed, (iii) the head of the household declares to have worked for the family business, and (iv) the head has a professional or managerial occupation. However, as I show in section 1.2, the magnitude of this decline does not significantly depend on the particular definition of entrepreneurs.

grew only by 10 log points. This suggests that newer entrepreneurs are selected from more productive workers.

The second contribution of this paper is to develop a quantitative model of entrepreneurial choice with two distinct skill groups to study the contribution of the skill-biased technical change and the decline in the relative cost of capital to the observed decline in entrepreneurship. In the model, a large number of heterogeneous individuals decide each period whether to be a worker or an entrepreneur conditional on their skill type, entrepreneurial ability, and assets. Additionally, entrepreneurs can borrow to increase the scale of their business but are subject to a collateral constraint.

The mechanism of my model is simple and works through the equilibrium effect of productivity improvements on profits and wages. As workers become more productive and capital becomes cheaper, both wages and profits increase for all entrepreneurs in the economy, existing and potential. However, because of the complementarity between capital and high skill labor, entrepreneurial profits for the marginal entrepreneur increase less that the wages she would obtain as a worker. This reduces individuals' incentives to run a business, thereby generating a decrease in the share of entrepreneurs. Yet, consistent with my empirical evidence, those individuals that do become entrepreneurs are increasingly more productive, raising average entrepreneurial productivity. Furthermore, the remaining entrepreneurs obtain larger profits because the workers they hire are more productive and capital is less costly.[6]

I calibrate the model to account for several salient features of the US economy in the mid-1980s, such as the share of entrepreneurs, the proportion of high and low skill entrepreneurs, and the wage skill premium. Then, I study the equilibrium transition of my modeled economy generated by three aggregate trends which have affected both entrepreneurial profits and the returns of high skill workers over the last three decades, each of which I consider as exogenous. The first is the skill-biased technical change, which refers to improvements in technology that have increased the productivity of high skill workers (Krueger, 1993). The second is the investment-specific technical change which induces a decrease in the relative cost of capital goods (Greenwood, Hercowitz, and Krusell, 1997). Notice that, if capital is more complementary to high skill labor than to low skill labor,

[6]My results are consistent with Michelacci and Schivardi (2016) that document a rapid increase in the profits for entrepreneurs relative to wages, especially at the highest educational level.

a decrease in the relative cost of capital goods increases the demand for high skill labor, generating an increase in the returns to high skill workers (Krusell et al., 2000). The third is the increase in the population share of college graduates observed in the United States over the last 30 years that has raised the supply of high skill labor. I take the decline of the cost of capital goods and the increase in the supply of high skill labor directly from the data, whereas the skill-biased technical change is calibrated to match the increase in the college wage premium observed in the United States between 1985 and 2015.

The main finding of this paper is that a standard model of entrepreneurial choice with the aforementioned trends can account for most of the decline in entrepreneurship observed in the United States between 1985 and 2014. In my modeled economy, the share of entrepreneurs drops 3.8 percentage points, almost all of the 3.9 percentage points decline observed in the data. I then decompose the contribution of each trend. I find that the skill-biased increase in productivity explains half of the reduction in the share of entrepreneurs in the US population, whereas the other half is equally explained by the decrease in the cost of capital goods and the increase in the supply of high skill labor. I find similar results when I decompose the time series of the transition rate into entrepreneurship implied by the model.

In the last part of this paper I consider a simple input cost subsidy that aims to bring the entry rate of new entrepreneurs in 2014 to the level observed in 1985. This subsidy relaxes entrepreneurs' borrowing constraint–the only source of inefficiency in my model–inducing more individuals to start a firm. I find this policy generates a sizable increase in the share of entrepreneurs and an increase in output and productivity. Specifically, relative to the baseline stationary economy, the share of entrepreneurs increases by 2.42 percentage points, output grows by 4.0%, and productivity grows by 9.2%. The increase in output and productivity stems from two factors: first, the reallocation of resources to existing entrepreneurs that can operate their firms closer to the optimal scale, and second, the entrance of new entrepreneurs that were borrowing constrained in the unsubsidized equilibrium. Welfare also improves, with the group of high skill entrepreneurs experiencing the largest increase. The cost of this subsidy, however, is quite substantial, amounting to 3.2% of the GDP.

Literature Review

This paper relates to several areas of research. First, my paper contributes to the growing literature on the decline of firm creation and dynamism experienced by the US economy in recent decades. Hyatt and Spletzer (2013) document a decline of several measures of job market dynamism such as job creation, job destruction, and job-to-job flows using firm- and individual-level data. Davis and Haltiwanger (2014) show that the decline of job reallocation rates had harmful effects on employment growth even before the Great Recession. Furthermore, several studies by Reedy and Strom (2012), Hathaway and Litan (2014a), Decker et al. (2014, 2016), Pugsley and Sahin (2014), Gourio et al. (2016), and others have documented a decrease in the share of activity accounted for by new and small firms. These papers show that this decline is not limited to a particular industry or geographical area. This suggests that structural factors are responsible for the decline in the pace of firm creation experienced by the US economy. My research complements these studies by using individual-level data to show that the decline in the startup rate has been accompanied by a fall in the share of entrepreneurs in the population.

Recent studies have postulated that an aging population is partly responsible for the decline in entrepreneurship. For instance, Liang, Wang, and Lazear (2014) exploit cross-country variation to quantify the importance of differences in the age distribution for entrepreneurship. Similarly, Hathaway and Litan (2014b) and Karahan et al. (2016) use differences in population growth across states in the United States to explain the decrease in the startup rate. However, differences in the decrease in the proportion of entrepreneurs across different age groups would suggest that additional factors are also important drivers of the decline in the startup rate.

Another possible explanation for the decline in firm entry is an increasing cost of regulation that affects existing and new entrepreneurs (Davis and Haltiwanger, 2014). My model captures a regulatory burden by imposing a cost on firm creation, which I can use to quantify the effect of an increase in the entry cost on entrepreneurship. Specifically, I ask what entry cost is necessary to reduce the share of entrepreneurs observed in 1985 to the level in 2014, absent any other change in the economy. Comparing these two stationary economies, I find that an entry cost that is seven times the cost in the initial stationary equilibrium, representing 2% of total output, is necessary to generate the drop in entrepreneurship observed in the data. Furthermore, an increasing cost of firm creation is unable to explain the differential decrease in the share of entrepreneurs between high and low skill individuals.

My paper is also related to the literature on entrepreneurial choice and its macroeconomic consequences. See Quadrini (2011) and Buera, Kaboski, and Shin (2015) for excellent reviews on the subject. Quadrini (2000) and Cagetti and De Nardi (2006) build models with an entrepreneurial choice to analyze the relation between entrepreneurs and the level of wealth inequality observed in the US economy. However, these papers study stationary economies and do not consider how aggregate trends, such as the ones that I consider in my quantitative analysis, affect the population share of entrepreneurs and wealth accumulation over time.

The work of Jian and Sohail (2017) is closely related to my paper. The authors document a decline in the transition rate into self-employment using a matched sample of the Current Population Survey (CPS). They show that the decline in the transition rate is stronger among college graduates. Then they study the effects of a skill-biased increase in productivity on the share of entrepreneurs in the context of a static occupational choice model. I depart from these authors in at least three aspects: First, I study the decline in the fraction of entrepreneurs and the transition into entrepreneurship considering a more comprehensive definition of entrepreneurship. Second, I consider a general equilibrium model that incorporates several degrees of heterogeneity and is consistent with the increase in the skill premium and with the extent and increase of wealth inequality observed in the United States. This is crucial because wealth accumulation is an important determinant of the transition into entrepreneurship. Third, in my quantitative exercise, I consider not only the direct effect of the skill-biased technical change, but also the indirect effect of a decline in the relative cost of capital and the increasing share of high skill workers in the population, both of which are absent from the analysis of Jian and Sohail (2017).

The rest of the paper is organized as follows. Section 1.2 shows evidence about the decline in the share of entrepreneurs in the US population, for different education and age groups. The evidence of section 1.2 motivates the entrepreneurial choice model that I describe in section 1.3. Section 1.4 presents the results of the model, and section 1.5 presents the policy and welfare analysis. Section 1.6 concludes.

1.2 Measuring Entrepreneurship

In this section, I show that the US economy has experienced a steady decline in the fraction of the population participating in entrepreneurial activities. Moreover, this decline is stronger among individuals with a college degree or more. I also show that the transition rate into entrepreneurship, that is, the share of wage workers that start a business in the following year, has also fallen in recent decades.

1.2.1 Data and Definitions

I this subsection I describe the sample that I use to study the evolution of the share of entrepreneurs in the population and the transition into entrepreneurship. My main data source is the Panel Study of Income Dynamics (PSID), which is a nationally representative survey that was conducted annually in the United States from 1968 to 1997, and every two years thereafter, on a sample of approximately 5000 families.[7] My results are based on a sample of heads of households from 1985 to 2015. The sample includes information on gender, income, education attainment, self employment status, and whether the household owns a business. The sample comprises those who are in the labor force and between 22 and 60 years old (both ends of the range included). All the statistics presented below are calculated using sample weights. Appendix A.1 describes in full detail the sample selection and variable construction.

Defining who is an entrepreneur is difficult and the empirical literature on entrepreneurship offers little consensus about which households or individuals should be classified as such.[8] Henceforth, most of the results in this section refer to four classifications of entrepreneurs that encompass the different alternatives considered in the literature. The PSID provides several questions that can be used to classify individuals by their entrepreneurial status.

[7] In this study I focus on the Survey Research Center sample (SRC). However, the main conclusions of the empirical section remain almost unaltered if we include the Survey of Economic Opportunities sample and use the proper weights.

[8] Evans and Leighton (1989) considers as entrepreneurs those that are self-employed, Hurst and Lusardi (2004) all those households that own a business, whereas Gentry and Hubbard (2004) defines as entrepreneurs all those business owners with businesses with a total market value of $5,000 or more. Quadrini (2000) considers both, business owners and self-employed as entrepreneurs. Cagetti and De Nardi (2006) and Michelacci and Schivardi (2016) define entrepreneurs as those self-employed business owners that have an active management in the firm.

In my analysis, I use four questions. The first is, "Did you (or anyone else in the family there) own a business at any time in (year) or have a financial interest in any business enterprise?" The second question is, "On your main job, are you (head) self-employed, are you employed by someone else, or what?" Third, starting in 1984 the heads of household are asked, "Did you (head) put in any work time for this business in (year)?" Finally, I use the occupation of the head of the household. Using this information, I separate households into four different groups. The first group considers all households who are "business owners" (those who answered affirmatively to the first question). Between 1985 and 2014, this group represented an average of 16.7% of all households in the United States. Second, I consider business owners that declare having worked for their businesses during the previous year, denoted as "active business owners". These households account, on average, for 14.8% of the population. The third group considers households who are business owners, worked for their businesses, and whose head is self-employed, that is, "self-employed business owners". These households represented an average of 10.0% of the population between 1985 and 2014. Finally, I define as "entrepreneurs" those self-employed business owners who have a managerial or professional occupation. These households, which are closer to the definition of an entrepreneur in my model economy, represented an average of 6.0% of the population between 1985 and 2014. Table 1.1 reports the average number of households in each class and their average share in the population. Table 1.1 also shows the share of entrepreneurs at the start and end of the sample. Notice that independent of the definition used, the share of households participating in entrepreneurial activities has declined substantially, between 3.5% and 5% during the last 30 years. Appendix table A.1 reports additional characteristics of the sample and within the different classifications of entrepreneurs.

1.2.2 The Declining Share of Entrepreneurs

In this subsection, I document a decline in the proportion of entrepreneurs in the US population and the drop in the share of households transitioning into entrepreneurship between 1985 and 2014. The left panel of figure 1.1 shows the substantial drop in the population share of individuals participating in entrepreneurial activities. For instance, in 1985, 16% of households in the United States were active business owners, whereas in 2014 only 12% of households were classified as such. Similarly, in 1985, 7.8% of all households

Table 1.1: Proportion of Entrepreneurs in the Population

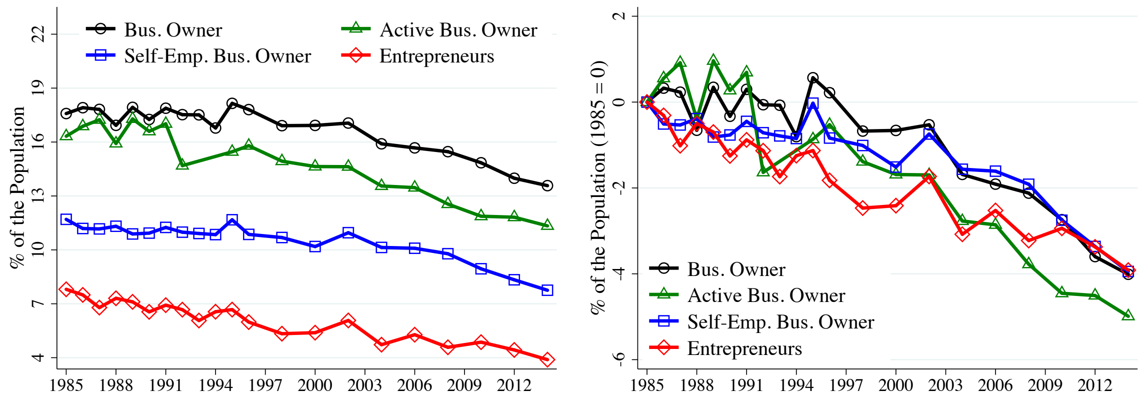
		(1)	(2)	(3)	(4)
Years	Obs.	Business Owner	Active Bus. Owners	Self-Emp. Bus. Owner	Entrepreneurs
[1985 – 2014]	3.664	16.73%	14.84%	10.50%	6.0%
1985	2.902	17.58%	16.32%	11.69%	7.80%
2014	4.140	13.57%	11.33%	7.78%	3.90%
Δ(2014 – 1985)		-4.01%	-5.99%	-3.91%	-3.90%

Note: Table 1.1 shows the average proportion of entrepreneurial households for four different classifications. Business owners are households whose head declares that he or another member of the household owns a business. Active business owners are households whose head declares having worked for the family business in a given year. Self-employed business owners are households classified as active business owners whose head declares being self-employed in his or her main job. Finally, entrepreneurs are households classified as self-employed business owners whose head has a managerial or professional occupation. The first row shows the average proportion within each group between 1985 and 2014. The second and third rows show the share of entrepreneurs in 1985 and 2014, respectively. The last row shows the change between 1985 and 2014 (differences are statistically significant at the 1% level of confidence).

can be classified as entrepreneurs. This figure was 3.9% in 2014. To better appreciate the decline in the rate of entrepreneurship across different definitions, the right panel of figure 1.1 shows the time series of the share of entrepreneurs rescaled by the level in 1985.[9]
The top panels of figure 1.2 show the time series of the fraction of entrepreneurs separating the population into two education groups. Two patterns are worth noticing. First, the share of entrepreneurs among college graduates is significantly larger than the share of entrepreneurs among high school graduates and dropouts. This is true independent of the definition of entrepreneur that one uses. Second, although both groups have experienced a decline in the share of entrepreneurs, the drop is steeper for the group of households with college education. This is more clearly shown in the bottom panels of figure 1.2, which display the share of entrepreneurial households rescaled to the level in 1985: the group of households whose head is a college graduate experienced a decline of roughly 7 percentage

[9]In this section, I calculate the fraction of entrepreneurs on a sample of heads of household that considers those that did and did not work for a wage in the corresponding year. Dropping the later group of observations–mostly unemployed heads of household–changes the level of the share of entrepreneurs in the population (it reduces the denominator). However, the declining trend remains almost the same, as I show in appendix figure A.4.

Figure 1.1: Share of Entrepreneurs



Note: The left panel of figure 1.1 shows the proportion of entrepreneurial households in each year for different classifications. From top to bottom, the slopes are -0.13, -0.20, -0.10, and -0.12, respectively. All slopes are statistically significant at the 1% level of confidence. The right panel shows the proportion of entrepreneurial households normalized by the value in 1985. See notes in table 1.1 for more details on the classification of entrepreneurial households.

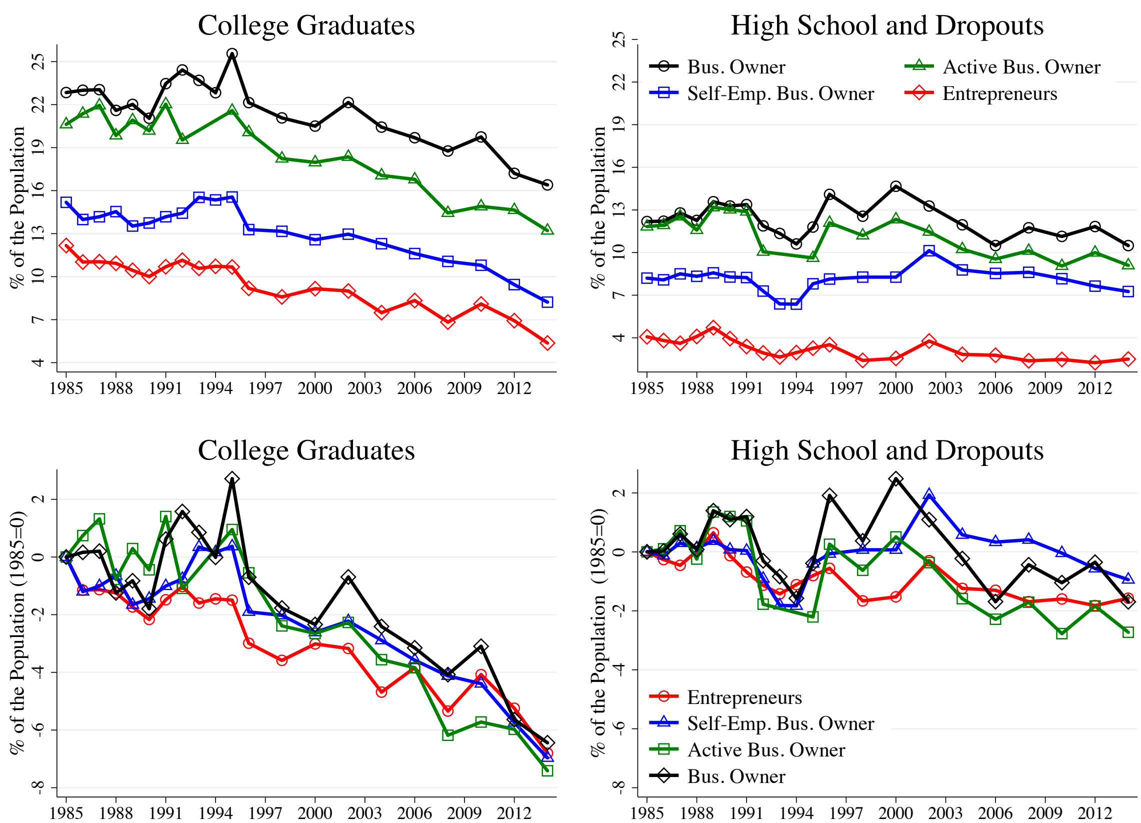
points between 1985 and 2014 (a 50% decline), whereas the decline for the group with a high school diploma or less is about 2 percentage points (a 20% decline).

Transition into Entrepreneurship

The decline in the share of entrepreneurs has been accompanied by a decrease in the rate at which workers decide to start new businesses. To illustrate this, I calculate the transition rate into entrepreneurship across the population and over the years. In order to have a more direct comparison between the different classifications of entrepreneurial households, I measure the transition rate as the share of the population that is neither a business owner nor self-employed in year t, but transitions into one of the four classifications in year $t+2$.[10] Figure 1.3 shows that the transition rates have declined for each classification since 1985. The drop is substantial: in 1985, 8.1% of the households that did not own a business or were self-employed started a business two years after. This figure was only 4.2% in 2014, which implies a decline of 50% in the transition rate. I find a similar drop in the transition rates for the rest of the definitions of entrepreneurial households (see right panel of figure

[10] I construct two-year transition rates to accommodate the biannual waves of the PSID after 1997.

Figure 1.2: Share of Entrepreneurs by Education Groups



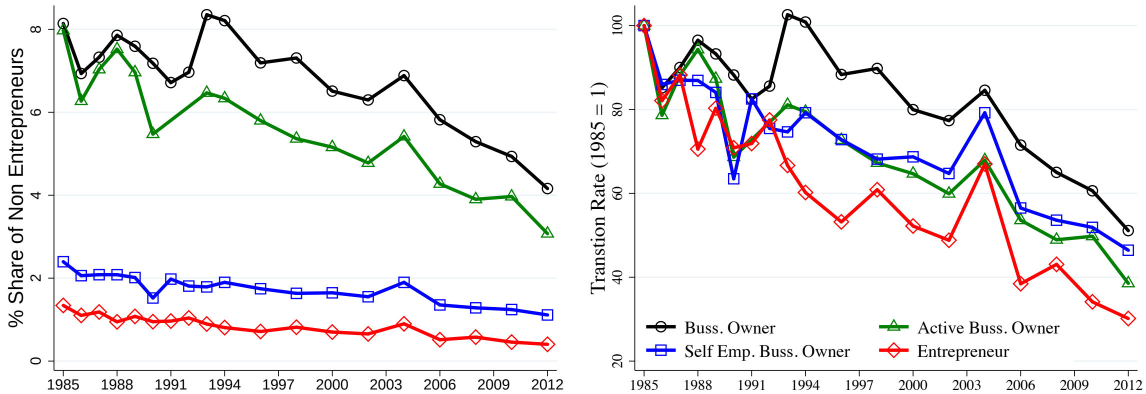
Note: Figure 1.2 shows the proportion of entrepreneurs within education groups. Individuals with some college are not considered. The bottom plots show the same statistics rescaled to their corresponding levels in 1985. See notes in table 1.1 for more details on the classification of entrepreneurial households.

1.3). The exit rate out of entrepreneurship, that is, the share of active entrepreneurs in period t that transitioned to being wage workers in $t+2$, does not show any particular trend between 1985 and 2014.[11]

Next, I use the panel dimension of the PSID to study how the characteristics of the households that start new businesses have changed over time. Specifically, I look to the wage level of entrepreneurs before they started their business. Arguably, workers with higher wages are more skilled than workers with lower wages. Therefore, an increase in the wage

[11]Using data from the Survey of Consumers Finances, Michelacci and Schivardi (2016) show that the exit rate from entrepreneurship has declined since 1989.

Figure 1.3: Transition rate to Entrepreneurship



Note: Figure 1.3 shows the proportion of households that are neither business owners nor self-employed in period t that are classified as entrepreneurial households in period $t + 2$ for the different definitions of entrepreneurship. See notes in table 1.1 for additional details of the definition of entrepreneurial households.

of the households that transition into entrepreneurship might be indicative that new entrepreneurs are more skilled and expect higher future profits, since they gave up higher earnings to start their firms. Hence, the decline in the share of entrepreneurs would have been accompanied by a selection of more talented individuals. To see whether this is the case, I consider a sample of male, heads of household who are neither self-employed nor business owners in year t (wage workers in period t). For each individual, I measure recent labor earnings as the average of total labor earnings between years t and $t - 2$.[12] I divide the sample into two groups: those with a high school diploma or less and those with at least some college studies.[13] Then, I calculate the average recent labor earnings within the group of individuals that become business owners in $t + 2$ (switching households) and within those that stay as workers (non-switching households).

Figure 1.4 shows that, within the group of individuals with some college, the average wage of those who became entrepreneurs grew faster than the average wage of individuals who remained as workers. The difference in the growth rate of earnings is both economically

[12]I calculate recent labor earnings to reduce business cycle variations which can heavily affect workers at the bottom of the skill distribution.

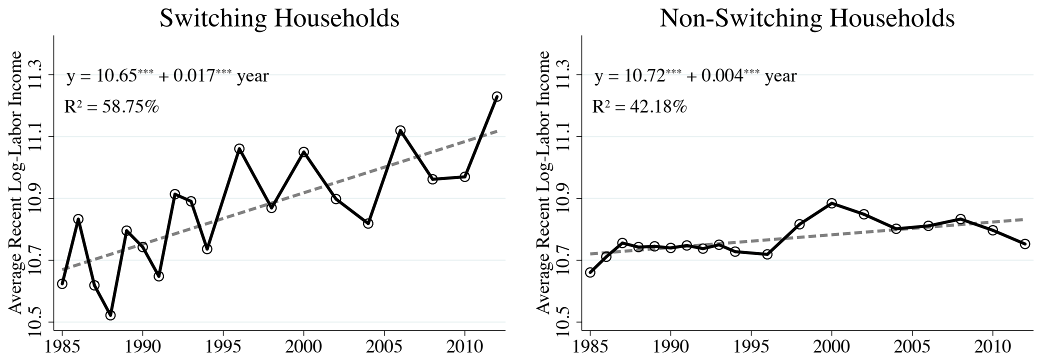
[13]The results are stronger if one includes workers with some years college in the first group strengthens my results.

and statistically significant (at the 1% level of confidence): the average recent earnings of workers that became entrepreneurs grew 1.7% per year between 1985 and 2014, accumulating more than 35% increase in three decades. On the other hand, the average earnings for those that remained as workers grew less than 0.4% on average during the same period of time, accumulating roughly a 10% increase. This suggests that new high skill entrepreneurs are increasingly selected from a pool of workers with higher average wages and are therefore more talented. In contrast, the growth rate of wages did not differ significantly between switching and non-switching workers within the group of individuals with a high school diploma or less. Figure 1.5 shows that the average recent earnings for less educated households transitioning to entrepreneurship decreased during the sample period, whereas earnings increased less than 0.1% per year for individuals that remained as workers.[14]

Several studies have shown the importance of wealth and borrowing constraints to explain entrepreneurial choices. In particular, the transition rate into entrepreneurship varies greatly across the wealth distribution and between individuals of different education groups (see, for instance, Hurst and Lusardi (2004) and Mondragón-Vélez (2009)). Similarly to other studies, I find significant differences in the wealth accumulation of workers before they start their own business relative to those workers that do not transit to entrepreneurship. Specifically, the median wealth of workers transitioning into entrepreneurship in the following period is 30% higher than the wealth of workers that stay as such. These differences increase if one considers higher percentiles of the wealth distribution (the differences are 37% and 46% at the 90th and 95th percentiles of the wealth distribution respectively).

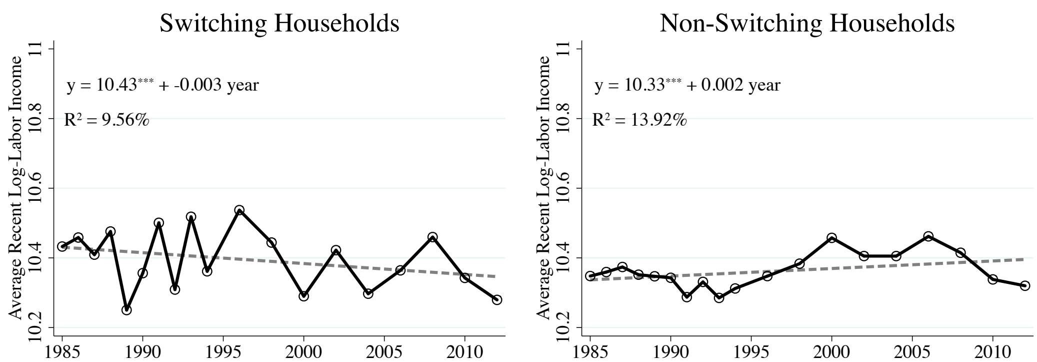
[14]Figure A.3 in appendix A.4 shows the results using a pooled sample of entrepreneurs. In such case, the growth rate of recent earnings for switching workers grew an average of 1.3% per year between 1985 and 2014 but only 0.5% for non-switching workers. In appendix A.4 I also show that these results are quite robust and do not change much if we consider wages and salaries instead of total labor earnings (figure A.7), if we consider current labor earnings (figure A.8), or if we look at the 50th percentile of the labor earnings distribution (figure A.9). The differences in the growth rate of earnings between those that switch to being business owners and those that remain as workers tend to vanish at higher levels of the recent income distribution. This can be seen in figure A.10, which shows the evolution of the 90th percentile of the recent labor income distribution.

Figure 1.4: Labor Income for Workers with Some College or More



Note: Figure 1.4 shows the average log of recent labor earnings for a sample of men, heads of household, who are neither business owner nor self-employed in year t and have some college studies or more. The left panel shows the average recent earnings within the group of households that become business owners in year $t+2$, whereas the right panel shows the same statistics for individuals that remain as workers in period $t+2$. The difference in the slope between the left and right panels is statistically significant at the 1% level.

Figure 1.5: Labor Income for Workers with High School Diploma or Less



Note: Figure 1.5 shows the average log of recent labor earnings for a sample of men, heads of household, who are neither a business owner nor self-employed in year t and have a high school diploma or less. The left panel shows the average recent earnings within the group of households that become business owners in year $t+2$, whereas the right panel shows the same statistics for individuals that remain as workers in period $t+2$. There is no statistical difference between the slopes in the left and right panels.

1.2.3 Evidence from the Survey of Consumer Finances

In this subsection, I present additional evidence on the decline in the share of entrepreneurs in the United States using information from the Survey of Consumer Finances. The SCF is a nationally representative survey conducted every three years since 1983. The SCF over samples wealthy households, which are more likely to be entrepreneurs (Cagetti and De Nardi, 2006), allowing for a more precise measure of the share of entrepreneurs in the population. Besides collecting information on household assets and business ownership, the SCF asks whether the head or another member of the family has an active management role in any of the businesses he or she owns. The main disadvantage of the SCF is that it does not follow individuals over time.

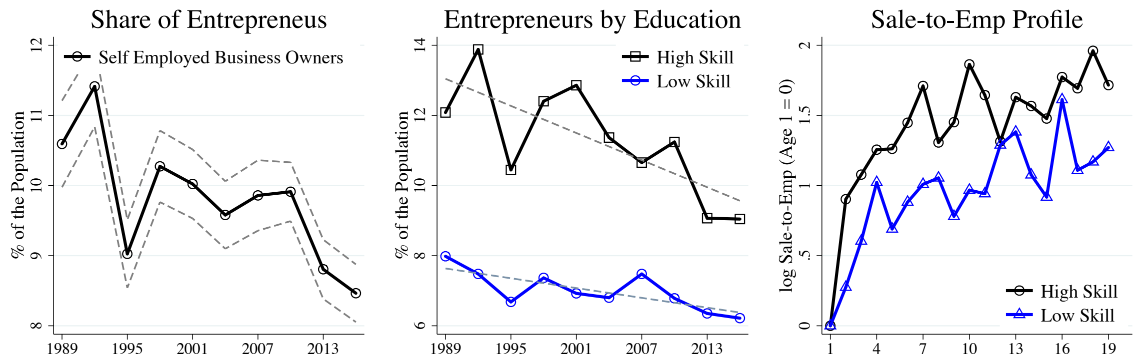
I use information from the SCF from 1989 to 2016. Following the definitions of the previous section, I define as self-employed business owners those households whose head or spouse has an active management role in a business owned by the family and whose head is self-employed.[15] These households represented an average of 9.2% of the total population between 1989 and 2016. The left panel of figure 1.6 shows that the share of self-employed business owners in the SCF declines over time, from 10.1 in 1989 to 7.8 in 2016. Separating the sample by education groups, I find that the decrease in the share of entrepreneurs is concentrated among households whose head has at least some college education (center panel of figure 1.6).[16]

The firms of high skill entrepreneurs are more profitable than the firms of low skill entrepreneurs. The right panel of figure 1.6 displays the (average) age profile of the log real sales-to-employment ratio after controlling for industry and year fixed effects, and by the gender and age of the entrepreneur. For better comparison, I have normalized each

[15] See Appendix A.5 for additional details on the construction of the SCF sample.

[16] Michelacci and Schivardi (2016) also use the SCF and suggest that the share of entrepreneurs is stable between 1989 and 2013. The difference is mainly due to the sample selection. Similar to Hurst and Lusardi (2004), I consider individuals between 22 and 60 years old, whereas Michelacci and Schivardi (2016) consider the entire sample of heads of household, independent of their age. Appendix figure A.15 shows the share of entrepreneurs with and without this age restriction (left panel) and within each age group (center panel). The share of entrepreneurs within the group of individuals of more than 60 years old has increased over time. Since this group has increased its population share, it is not surprising to find that the share of entrepreneurs is more stable if these individuals are considered in the sample: between 1989 and 2016, the share of entrepreneurs declines 1.55 percentage points among individuals between 22 and 60, but only 0.5 percentage points considering the entire sample. In this paper, I focus on the 22 to 60 range because these individuals are more likely to switch between workers and entrepreneurs and are more inclined to start new businesses. The right panel of appendix figure A.15 shows that the share of startup entrepreneurs (entrepreneurs whose main business is one year old or less) among individuals 22 and 60 averages 17% between 1989 and 2016 and has been declining over time, much like the evidence presented by Pugsley and Sahin (2014) using firm-level data, whereas the share of startups within the group of entrepreneurs older than 60 is smaller (average of 5%) and has increased over the last 25 years.

Figure 1.6: Characteristics of Entrepreneurs in the SCF



Note: The left and center panels of figure 1.6 display the time series of the share of self-employed business owners in the SCF. High skill entrepreneurs are heads of household with some college studies or more. Low skill entrepreneurs are heads of households with a high school diploma or less. Self-employed business owners are households that own a business, declare having an active management role, and whose head is self-employed. The slopes in the center plot are statistically different at the 5% level. The right panel shows the average sales-to-employment ratio. Each point is the value of the age fixed effect on a regression of the log sales-to-employment on industry, year, and gender fixed effects, and a quadratic in the age of the entrepreneur. All monetary values are expressed in 2012 US dollars. See Appendix A.5 for additional details on the construction of the SCF sample.

profile by the sales-to-employment ratio in the first year. The firms of high and low skill entrepreneurs generate a similar amount of dollars per worker in their first year of operation (an average of \$16,400 per employee in the case of high skill entrepreneurs relative to \$15,500 for low skill entrepreneurs), but the firms of high skill entrepreneurs grow much faster than the firms of low skill entrepreneurs. The average sales-per-employee ratio of the firms managed by low skill entrepreneurs grows by 21% in the second year of operation and grows by 80% when the firm has reached the age of five. This increase is more substantial among the firms managed by high skill entrepreneurs: in the second year, these entrepreneurs are selling 80% more per worker than in their first year, and after five years of operation, the sales-to-employment ratio is 150% higher than in the first year. The differences are also economically significant. The median high skill entrepreneur running a firm that has been operating for five years generates \$77,812 per worker, while a firm of the same age but owned by a low skill entrepreneur generates \$28,874 per worker. I do not find important differences between average employment size of firms of high and low skill entrepreneurs after I have controlled for industry.

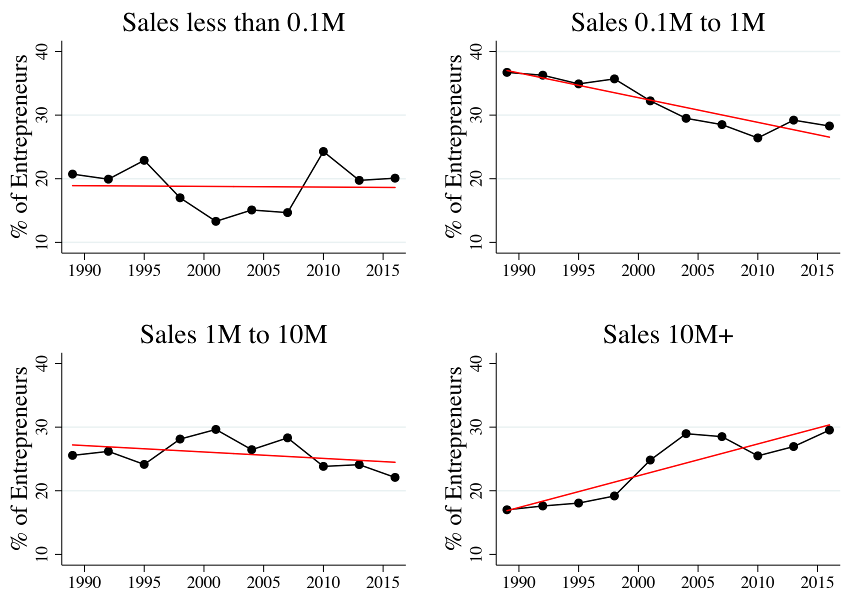
Using the SCF one also can study in more detail the changing characteristics of entrepreneurs and their firms. For instance, one would like to know whether the decline in the share of entrepreneurs is concentrated among those managing small, medium, or large firms. This would give us a sense of how important is the decrease in the share of entrepreneurs. To see this, I separate the sample of entrepreneurs in the SCF into four size categories, measured by the amount of sales (expressed in 2012 dollars). I consider three cut-offs, one at 0.1 million dollars, a second at 1 million dollars, and the third at 10 million. Then, I calculate the share of entrepreneurs within each of these groups. Figure 1.7 shows that the decline of the proportion of entrepreneurs has been accompanied by a shift in entrepreneurs' size distribution. In particular, the share of the smallest group has stayed constant over the years, the share of middle-sized entrepreneurs, with sales between 0.1M and 1M, has declined, whereas the group of entrepreneurs selling more than 10 million has increased substantially over time. Importantly, this change in the sales-size distribution of entrepreneurs is mostly explained by changes in the distribution of high skill entrepreneurs, which represent a disproportionate share of the group or large entrepreneurs: high skill entrepreneurs represent 80% of all entrepreneurs selling more than 10M dollars per year, but less than 20% in the group of entrepreneurs selling less than 0.1M. Furthermore, the sales-size distribution among low skill entrepreneurs almost did not change over the sample period as shown in Appendix figure A.16.[17]

In summary, I have shown that the US economy has experienced a decline in the proportion of households involved in entrepreneurial activities. The decline is stronger among more skilled households. I also find a decline in the transition rate into entrepreneurship and an increase in the selection of entrepreneurs with higher wages. Building on this evidence, the next section presents an equilibrium model that is consistent with the decline in the share of entrepreneurs, with the differential decrease in the share of entrepreneurs across education groups, and with the increase in the selection of more talented workers to start new firms.[18]

[17]This results are robust to other classification in terms of sales size and if I use employment instead of sales as a measure of entrepreneur's firm size.

[18]For further robustness, appendix A.1.2 presents additional evidence on the declining share of entrepreneurs using a sample drawn from the CPS.

Figure 1.7: Size Distribution of Entrepreneurs



Note: The figure 1.7 shows the evolution of the share of entrepreneurs for different size classifications. All monetary values are expressed in 2012 US dollars. See appendix A.5 for additional details on the construction of the SCF sample.

1.3 The Model

1.3.1 Households and Production

This section describes the model I use to study the effect of technical change and the increase in the supply of skilled labor on the decision to become an entrepreneur.

Demographics

Consider an economy with a continuum of individuals of measure one. In each period, there is a proportion H_t of high skill individuals and a proportion L_t of low skill individuals. An individual dies with probability $(1-\chi)$, in which case her offspring enters the model carrying the same skill type of her parents with probability ζ_s, with $s \in \{H, L\}$. She also inherits the assets bequeathed by her parent and her parent's business in the case the parent dies as an entrepreneur.

Preferences and Discounting

Each individual values consumption by means of the utility function $c_t^{1-\sigma}/(1-\sigma)$ and supplies one unit of labor inelastically. Individuals discount future streams of utility at the rate $\beta < 1$, and the utility of their offspring by a proportion $\beta\eta$ with $\eta \in [0, 1]$.

Production Technology

In each period, an individual decides whether to work or to become an entrepreneur (labor is indivisible).[19] If the individual decides to be a worker, she receives an income of $\omega_{s,t}y_t$, where $s \in \{H, L\}$, y_t is an idiosyncratic, positively autocorrelated shock, and $\omega_{s,t}$ is the wage of a worker of type s in period t. A worker cannot borrow but can save in a riskless asset, a_t, with return r_t.

If the individual chooses to be an entrepreneur, she gains access to a productive technology that uses four different factors: Her own entrepreneurial ability, low skilled labor, $n_{L,t}$, high skilled labor, $n_{H,t}$, and capital, k_t. All entrepreneurs produce the same homogeneous good. Entrepreneurial ability has two components: a fixed part, denoted by θ_s, which depends on the skill type of the individual, and an idiosyncratic part, z_t, which is positively autocorrelated and independent of y_t. The production technology available to the entrepreneur is $z_t\theta_s \left[f\left(n_{H,t}, n_{L,t}, k_t\right)\right]^{\gamma}$, where $\gamma < 1$ is the span-of-control parameter that determines the degree of decreasing returns to scale, and hence, the returns to entrepreneurial ability (Lucas, 1978). The function $f\left(n_{H,t}, n_{L,t}, k_t\right)$ is given by

$$f\left(n_{H,t}, n_{L,t}, k_t\right) = \left[\psi\left(\tau\left(A_{H,t}n_{H,t}\right)^{\rho} + (1-\tau)\,k_t^{\rho}\right)^{\frac{\alpha}{\rho}} + (1-\psi)\,n_{L,t}^{\alpha}\right]^{\frac{1}{\alpha}}. \tag{1.1}$$

The value of ρ determines the elasticity of substitution between high skill labor and capital, and α determines the elasticity of substitution between the composite of capital and low

[19]In this model, the assumption of indivisibility of labor is important. Alternatively, one could assume that individuals can receive a wage and run a business at the same time. In such case, since firms profits are always positive and there is no uncertainty about firms returns, the individual does not face any tradeoff between be a worker or be an entrepreneurs. In this case, and increase of the productivity of high skill workers (or a decrease in the price of investment) results in an increase in the share of entrepreneurs. However, in such model, individuals are not entrepreneurs in the sense that I consider in this paper, but business owners that have a firm as an investment opportunity.

skill labor. The parameter τ determines the output share of capital whereas ψ determines the output share of labor. The value of $A_{H,t}$ captures a skill-biased change in productivity that directly affects the relative contribution of high skill workers to output. There is no fixed cost of production; however, creating a new firm implies a one period cost, κ. Notice this cost affects only individuals transitioning from wage worker to entrepreneur.

In reality, a large fraction of firms are not managed by individuals weighing the cost and benefit of running their own business or working in someone else's company. Therefore, as in Quadrini (2000) and Cagetti and De Nardi (2006), I model a second sector of production populated by a large number of homogeneous firms operating a constant returns to scale production technology given by

$$F\left(N_{H,t}, N_{L,t}, K_t\right) = A\left[\psi\left(\tau\left(A_{H,t}N_{H,t}\right)^{\rho} + (1-\tau)\, K_t^{\rho}\right)^{\frac{\alpha}{\rho}} + (1-\psi)\, N_{L,t}^{\alpha}\right]^{\frac{1}{\alpha}},$$

which I will refer to as the non-entrepreneurial sector. Both sectors produce the same good, and in both sectors capital depreciates at the rate δ.

Borrowing Constraint

Several papers have documented the importance of borrowing constraints to the decision to become an entrepreneur.[20] Here I assume that entrepreneurs need to buy capital and pay wages before revenues are realized. This captures that idea that an entrepreneur needs some working capital to run her business. To finance this working capital, entrepreneurs obtain an intraperiod loan with gross interest of $(1 + r_t)$ and total amount $p_{k,t}k_t+\omega_{H,t}n_{H,t}+\omega_{L,t}n_{L,t}$, where $p_{k,t}$ is the price of capital goods in terms of consumption.[21] The maximum amount of the loan is constrained by the wealth of the household. In particular, each entrepreneur faces a simple collateral constraint of the form

$$p_{k,t}k_t + \omega_{H,t}n_{H,t} + \omega_{L,t}n_{L,t} \leq \lambda a_t,$$

[20]See, for instance, Evans and Jovanovic (1989), Hurst and Lusardi (2004), or Mondragón-Vélez (2009).

[21]My baseline results do not depend on this particular form of the borrowing constraint. Appendix A.3.1 compares my quantitative exercise under a different collateral constraint.

with $\lambda \geq 1$.[22]

Exogenous Aggregate Processes

The economy is subject to three exogenous aggregate processes: investment-specific technological change that decreases the relative price of capital goods, $p_{k,t}$, an increase in the supply of high skill workers, H_t, and a skill-biased improvement in technology that increases the productivity of high skill workers, $A_{H,t}$. In my baseline exercise I assume there is no aggregate uncertainty and the time series of each of these processes are fully known by the individuals.[23]

The Problem of the Individuals

At the beginning of the period, an individual is characterized by her fixed skill type, $s \in \{H, L\}$, asset level, a_t, entrepreneurial ability, z_t, worker ability, y_t, and previous occupation, $d_t \in \{w_t, e_t\}$, where w identifies a worker and e an entrepreneur. To simplify the notation, name the vector of idiosyncratic states by $\Omega_t \equiv \{a_t, z_t, y_t, d_{t-1}\}$, and the distribution of individuals of type s in period t over idiosyncratic states by $\mu_{s,t}$ with $\mu_t \equiv \{\mu_{H,t}, \mu_{L,t}\}$. Denote the vector of aggregate states by $\Theta_t \equiv \{p_{k,t}, A_{H,t}, H_t\}$. Then, a $s-type$ individual solves

$$V_{s,t}\left(\Omega_t, \Theta_t, \mu_t\right) = \max\left\{V^w_{s,t}\left(\Omega_t, \Theta_t, \mu_t\right), V^e_{s,t}\left(\Omega_t, \Theta_t, \mu_t\right)\right\}, \tag{1.2}$$

[22] This type of borrowing constraint can arise from a limited enforcement problem as in Jermann and Quadrini (2012) and has been used in several other papers. See, for instance, Evans and Jovanovic (1989), Buera (2009) Buera and Shin (2013), Moll (2014), Guvenen, Kambourov, Kuruscu, Ocampo, and Chen (2015), among others.

[23] I assume that the price of capital goods is exogenously given. This is equivalent to modeling a third productive sector with a linear production technology that transforms consumption goods into capital goods. A decrease in the relative price of capital would result from an increase in this sector's productivity.

where $V^{w}_{s,t}(\Omega_t, \Theta_t, \mu_t)$ is the value of being a worker in period t and $V^{e}_{s,t}(\Omega_t, \Theta_t, \mu_t)$ is the value of being an entrepreneur. The value of being a worker is given by

$$V^{w}_{s,t}(\Omega_t, \Theta_t, \mu_t) = \max_{c_t, a_{t+1}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \left[\chi \mathbb{E}_{z_{t+1}|z_t, y_{t+1}|y_t} V_{s,t+1}(\Omega_{t+1}, \Theta_{t+1}, \mu_{t+1}) + \right.\right.$$
$$\left.\left. (1-\chi)\eta \sum_{j\in\{H,L\}} \zeta_{s,j} \mathbb{E} V_{s,t+1}(\Omega_{t+1}, \Theta_{t+1}, \mu_{t+1}) \right] \right\} \quad (1.3)$$

$$c_t + a_{t+1} \leq (1 + r_t(\Theta_t, \mu_t))\, a_t + \omega_{s,t}(\Theta_t, \mu_t)\, y_t,$$
$$a_{t+1} \geq 0,$$

subject to the laws of motion of y_t and z_t, the law of motion of the distribution of individuals over idiosyncratic states, $\mu_{t+1} = \Psi(\Theta_t, \mu_t)$, and the evolution of the aggregate states, Θ_{t+1}. In the problem of the worker described by equation (1.3), the first expectation is taken over the conditional distributions of z_{t+1} and y_{t+1}, and over the next period's distribution of individuals over idiosyncratic states, while the second expectation is taken over the unconditional distributions of z_{t+1} and y_{t+1}, and over the next period's distribution of idiosyncratic states.

The value of being an entrepreneur is given by

$$V^{e}_{s,t}(\Omega_t, \Theta_t, \mu_t) = \max_{c_t, a_{t+1}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \left[\chi \mathbb{E}_{z_{t+1}|z_t, y_{t+1}|y_t} V_{s,t+1}(\Omega_{t+1}, \Theta_{t+1}, \mu_{t+1}) + \right.\right.$$
$$\left.\left. (1-\chi)\eta \sum_{j\in\{H,L\}} \zeta_{s,j} \mathbb{E} V_{s,t+1}(\Omega_{t+1}, \Theta_{t+1}, \mu_{t+1}) \right] \right\}, \quad (1.4)$$

$$\pi_{s,t}(z_t, a_t) = \max_{n_{H,t}, n_{L,t}, k_t} \left\{ z_t \theta_s \left[f(n_{H,t}, n_{L,t}, k_t) \right]^{\gamma} - p_{k,t}(r+\delta)\, k_t - \right.$$
$$\left. (1 + r(\Theta_t, \mu_t))\left(\omega_{H,t}(\Theta_t, \mu_t)\, n_{H,t} + \omega_{L,t}(\Theta_t, \mu_t)\, n_{L,t}\right) \right\},$$

$$c_t + a_{t+1} + \mathbb{I}\left(d_{t-1} = w_{t-1}\right)\kappa \leq \left(1 + r\left(\Theta_t, \mu_t\right)\right) a_t + \pi_{s,t}\left(z_t, a_t\right),$$
$$p_{k,t}k_t + \omega_{H,t}\left(\Theta_t, \mu_t\right) n_{H,t} + \omega_{L,t}\left(\Theta_t, \mu_t\right) n_{L,t} \leq \lambda a_t$$
$$a_{t+1} \geq 0,$$

subject to the non-negativity constraints of factor demands, the laws of motion of y_t and z_t, the law of motion of the distribution of individuals over idiosyncratic states, $\mu_{t+1} = \Psi\left(\Theta_t, \mu_t\right)$, and the law of motion of the aggregate states, Θ_{t+1}. Here, $\mathbb{I}\left(d_{t-1} = w_{t-1}\right)$ is an indicator function which is equal to 1 if the individual was a worker in the previous period and is equal to zero otherwise. This function captures the assumption that the fixed cost of creating a firm is paid only by those individuals transitioning from worker to entrepreneur. The solution of the problem of the household is characterized by a $z-$threshold which depends on the individual's level of assets, the labor productivity, and the previous period's occupation.

The Problem of the Non-Entrepreneurial Sector

The problem of the non-entrepreneurial sector is simple and is given by

$$\pi_{c,t} = \max_{N_{H,t}, N_{L,t}, K_t} \Big\{ F\left(N_{H,t}, N_{L,t}, K_t\right) - p_{k,t}\left(r\left(\Theta_t, \mu_t\right) + \delta\right) K_t - \omega_{H,t}\left(\Theta_t, \mu_t\right) N_{H,t} - \omega_{L,t}\left(\Theta_t, \mu_t\right) N_{L,t} \Big\}, \quad (1.5)$$

subject to the non-negativity constraints of factor demands.

1.3.2 Equilibrium

Given an initial distribution μ_0 and an exogenous path of $\Theta_t = \{p_{k,t}, A_{H,t}, H_t\}_{t=0}^{\infty}$, a recursive competitive equilibrium in this economy is

- A time path for prices, $\{\omega_{H,t}\left(\Theta_t, \mu_t\right), \omega_{L,t}\left(\Theta_t, \mu_t\right), r_t\left(\Theta_t, \mu_t\right)\}_{t=0}^{\infty}$, and a sequence of distributions over idiosyncratic states $\{\mu_{t+1}\left(\Theta_t, \mu_t\right)\}_{t=0}^{\infty}$,

- a sequence of individual's policy functions $\left\{c_t^s\left(\Omega_t, \Theta_t, \mu_t\right), a_{t+1}^s\left(\Omega_t, \Theta_t, \mu_t\right), d_t^s\left(\Omega_t, \Theta_t, \mu_t\right)\right\}_{t=0}^{\infty}$, with $\left\{V_{s,t}\left(\Omega_t, \Theta_t, \mu_t\right)\right\}_{t=0}^{\infty}$, the associated value functions and $s \in \{H, L\}$,
- factor demands for the entrepreneurs, $\left\{k_t^s\left(\Omega_t, \Theta_t, \mu_t\right), n_{H,t}^s\left(\Omega_t, \Theta_t, \mu_t\right), n_{L,t}^s\left(\Omega_t, \Theta_t, \mu_t\right)\right\}_{t=0}^{\infty}$,
- demands of the non-entrepreneurial sector, $\left\{K_t\left(\Theta_t, \mu_t\right), N_{H,t}\left(\Theta_t, \mu_t\right), N_{L,t}\left(\Theta_t, \mu_t\right)\right\}_{t=0}^{\infty}$,

such that

- The policy functions, value functions, and factor demands solve the individual's problem given by (1.3) and (2.6),
- The factor demands of the non-entrepreneurial sector solve (1.5),
- Labor markets for high and low skill workers clear,

$$\int \left(1 - d_t^H\left(\Omega_t, \Theta_t, \mu_t\right)\right) d\mu_t^H\left(\Omega_t\right) = N_{H,t}\left(\Theta_t, \mu_t\right) + \sum_{s \in H,L} \int n_{H,t}^s\left(\Omega_t, \Theta_t, \mu_t\right) d_t^s\left(\Omega_t, \Theta_t, \mu_t\right) d\mu_t^s\left(\Omega_t\right), \quad (1.6)$$

$$\int \left(1 - d_t^L\left(\Omega_t, \Theta_t, \mu_t\right)\right) d\mu_t^L\left(\Omega_t\right) = N_{L,t}\left(\Theta_t, \mu_t\right) + \sum_{s \in H,L} \int n_{H,t}^s\left(\Omega_t, \Theta_t, \mu_t\right) d_t^s\left(\Omega_t, \Theta_t, \mu_t\right) d\mu_t^s\left(\Omega_t\right), \quad (1.7)$$

- Capital market clears,

$$\sum_{s \in H,L} \int a_t^s\left(\Omega_t, \Theta_t, \mu_t\right) d\mu_t^s\left(\Omega_t\right) = p_{k,t} K_t\left(\Theta_t, \mu_t\right) + \sum_{s \in H,L} \int \Bigg(p_{k,t} k_t^s\left(\Omega_t, \Theta_t, \mu_t\right) + \omega_{H,t}\left(\Theta_t, \mu_t\right) n_{H,t}^s\left(\Omega_t, \Theta_t, \mu_t\right) + \omega_{L,t}\left(\Theta_t, \mu_t\right) n_{L,t}^s\left(\Omega_t, \Theta_t, \mu_t\right) \Bigg) d_t^s\left(\Omega_t, \Theta_t, \mu_t\right) d\mu_t^s\left(\Omega_t\right). \quad (1.8)$$

A stationary competitive equilibrium is similarly defined but over a constant path of Θ_t, which implies a constant sequence of prices and distributions over idiosyncratic states.

The solution of the model requires an initial and a final steady state, and the complete transition path of aggregate states and factor prices, given an exogenous sequence of Θ_t. Appendix A.2 describes in detail the algorithm that I use to solve the model.

1.3.3 Calibration

This section describes the quantitative specification of the model. To maintain the tractability of the calibration, I take some parameters directly from the literature (e.g., the risk aversion or the depreciation rate), I calculate other parameters directly from the data (e.g., the parameters governing the labor income process), and I choose other parameters such that the stationary equilibrium matches several features of the US data in the mid 1980s. In this section, I also describe how I pin down the three exogenous time series (the price of capital, the share of high skill workers, and relative productivity of high skill workers) that I will incorporate into the model. In order to highlight the effects of the change in wages on entrepreneurs, I will assume, for the moment, that this is a small open economy and the interest rate is constant over time. In section 1.4.4, I show how my quantitative results change when considering a full-fledged general equilibrium model.

Frequency, Preferences, and Discounting

I set the time period to equal a year. I take a standard value of 2.0 for the coefficient of risk aversion. In the baseline case, I set $\eta = 1$ so parents are perfectly altruistic. I assume β to be 0.88, and a fixed interest rate of 3.0%.[24]

Demographics

I set the value of χ equal to 0.025 so that the average working life of an individual is 40 years. In the baseline calibration, I assume when an individual dies, the offspring inherits her parent's skill type with probability one, so $\zeta_{hh} = \zeta_{ll} = 1$.

[24] I choose the value of β equal to 0.88 to be consistent with my calibration in the general equilibrium case which I discuss in the robustness section.

Production and Capital Depreciation

Following Krusell et al. (2000), I assume that capital is more complementary to skilled than to unskilled workers. Hence, I set α equal to 0.401 and ρ equal to -0.495. The value of γ in the technology of the entrepreneur is equal to 0.88, as in Cagetti and De Nardi (2006), and the depreciation rate of capital, δ, is set at an annual value of 0.06.

Labor Income Shocks

I assume that the process of y_t is such that $\log y_{t+1} = \rho_y \log y_t + \sigma_{\epsilon,y}\epsilon_{y,t+1}$ where $\epsilon_{y,t+1}$ is distributed normal with mean zero and unitary variance. The values of ρ_y and σ_y are estimated using data from the PSID using a sample of workers for the period 1970 to 1996.[25] In the model, all households are subject to the same stochastic shocks. Therefore, I select a pooled sample of heads of household with valid labor income that are neither business owners nor self-employed in periods t and $t-1$. Then, I estimate the following equation:

$$\log w_{i,t} = \beta_0 + \beta_1 A_{i,t} + \beta_2 A_{i,t}^2 + \beta_3 A_{i,t}^3 + \rho_w \log w_{i,t-1} + \nu_{i,t}, \tag{1.9}$$

where $w_{i,t}$ is the total real labor earnings of the head of the household in period t.[26] The right-hand side includes a polynomial in age to control for life-cycle patterns. The estimated autocorrelation of earnings, ρ_w, is 0.73, and the standard deviation of $\nu_{i,t}$ is 0.53. Using these values for ρ_y and σ_y, I discretize the continuous process using the method developed by Tauchen (1986). The sample selection and additional estimation results are described in appendix A.1.1. Table 1.2 shows the fixed parameters chosen from the literature or calculated directly from the data.

Exogenous Aggregate Processes

In this section, I discuss how I choose the time series of the relative price of capital goods, $p_{k,t}$, the supply of high skill labor, H_t, and the productivity of high skill workers, $A_{H,t}$.

[25] After 1997 the PSID becomes biannual and it is not possible to calculate the one-year changes.

[26] Here, total labor income includes only wages and salaries, bonuses, tips, and commissions. The labor part of businesses income and the income from farm are excluded as I am considering in the sample individuals that work for a wage only and do not own a businesses neither in period t nor in period $t-1$.

Table 1.2: Fixed Parameters in the Benchmark Model

		Value	Source
Risk Aversion	σ	2.0	-
Prob. Dying	$1-\chi$	2.5%	Average working life of 40 years
Perfect Altruism	η	1	Perfect Altruism
Depreciation	δ	0.06	-
Capital-High Skill ES	ρ	-0.49	Krusell et al. (2000)
Capital-Low Skill ES	α	0.40	Krusell et al. (2000)
Span-of-Control	γ	0.88	Cagetti and De Nardi (2006)
Autocorrelation of y	ρ_y	0.75	PSID
Standard Dev. of ϵ_y	$\sigma_{\epsilon,y}$	0.53	PSID

Note: Table 1.2 reports the set of fixed parameters used in the baseline model.

The first two time series have obvious empirical counterparts, whereas the third needs to be pinned down using additional conditions.

For the time series of $p_{k,t}$ I use the quality-adjusted relative price of capital goods calculated by DiCecio (2009) and normalized to 1 in 1985.[27] Using this time series, the relative price of capital declined 55% between 1985 and 2015.[28] In my simulation, I will assume that from 2015 on, $p_{k,t}$ remains fixed at its 2015 level for the rest of the simulation. This is an extreme case and therefore, in the robustness section, I study how sensitive my results are to a different assumption on the time trend of the price of capital and the rest of the exogenous trends after 2015.

I equate the share of high skill workers in the model, H_t, to the fraction of individuals with a college degree or more calculated from a sample of heads of household between ages 22 to 60 years drawn from the CPS. The share of college graduates in this sample increased from 26% in 1985 to 39% in 2015. In my baseline results, I assume that the share of high skill workers remains at its 2015 level for the rest of the transition.

Finally, I need to choose the time series for the skill-biased technological progress, $A_{H,t}$. I select the time series of $A_{H,t}$ such that the increase in the skill premium implied by the

[27] DiCecio (2009) extrapolates the quality-adjusted price time series of Gordon (2007) to 2010 using the same techniques of Cummins and Violante (2002). I take the updated time series up to 2015 from FRED.

[28] Alternatively, one could use the measure of the price of capital goods calculated by the Bureau of Economic Analysis (BEA) to compute the relative price of capital goods. Using this measure, the relative price of capital declined 50% between 1985 and 2015. See appendix A.1.3 for additional details and a comparison of the relative price of investment calculated by DiCecio (2009), the time series using the BEA's data, and an additional measure that only considers equipment and software.

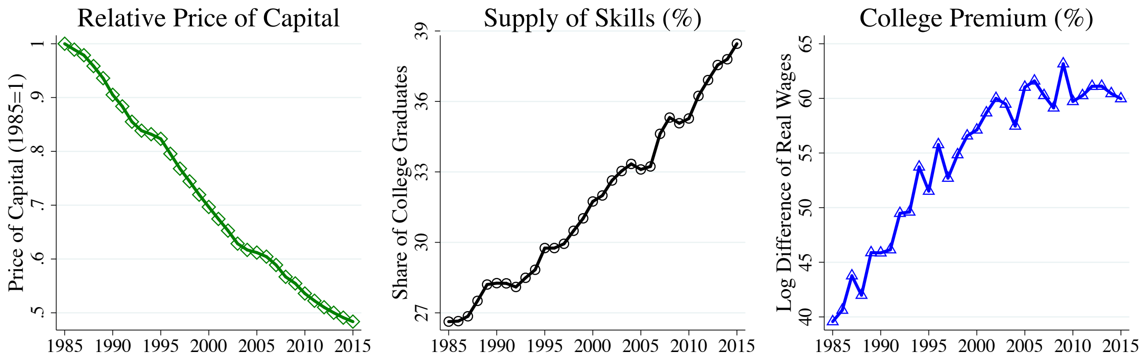
model (the log difference between the wage of high skill workers and the wage of low skill workers) matches the increase in the college premium observed over the last 30 years in the United States. I measure the college premium as the log difference of the real annual labor income of college graduates and the real annual labor income of high school graduates over a sample of workers from the CPS. Additional details of the construction of the skill premium are discussed in appendix A.1.2. Using this sample, the college premium increased from 39% in 1985 to 60% in 2015.

Because my model economy is subject to two additional aggregate trends (the price of capital and the supply of high skill workers), there is no clear mapping between the college premium implied by the model and the evolution of $A_{H,t}$. In particular, since high skill workers are more complementary to capital than low skill workers (recall $\alpha > \rho$ in the production function), the decline in $p_{k,t}$ and the rise in H_t will affect the skill premium in opposite directions. To solve this issue, I take a very simple approach: I fix the value of $A_{H,t}$ in the initial stationary economy to be equal to 1 and select the value of $A_{H,t}$ at the final stationary equilibrium so that my model matches a skill premium of 60% conditional on the values of $p_{k,t}$ and H_t in 2015. Then, the sequence of $\{A_{H,t}\}_{t=1}^{T}$ grows linearly between these two fixed points for 30 years. As I do with the other two aggregate exogenous processes, I assume that the value of $A_{H,t}$ is fixed for the rest of the transition. Figure 1.8 displays the time series of the relative price of capital goods, the share of college graduates, and the college premium in the data that I use to discipline the aggregate processes in my model.

Parameters Determined Jointly in Equilibrium

The parameters that are calibrated simultaneously with the equilibrium of the model are the factor shares in the production function, τ and ψ, the borrowing limit, λ, the entry cost, κ, and the parameters of entrepreneurial ability, θ_s and z_t. I normalize θ_H to 1, and I assume that $\log z_{t+1} = \rho_z \log z_t + \sigma_{\epsilon,z}\epsilon_{z,t+1}$. This leaves seven parameters that need to be calibrated jointly with the equilibrium of the model. I use these seven parameters to pin down the same number of moments generated by my model in 1985. I select this particular year because it is the first for which I have information about each of the moments that I seek to match. I normalize the relative price of capital goods, $p_{k,t}$, and the productivity of high skill workers, $A_{H,t}$, to 1 in 1985, and I set the share of high skill workers, H_t, to

Figure 1.8: Aggregate Processes



Note: Figure 1.8 displays the time series of the relative price of capital goods (left panel), the share of college graduates in the population (center panel), and the college premium (right panel). The relative price of capital goods (calculated by DiCecio (2009)) is normalized to 1 in 1985. The supply of college graduates is calculated over a sample of heads of household drawn from the CPS. The supply of college graduates in each year is the share of heads of household between ages 22 and 60 that have a college degree or more. The college premium is calculated as the difference between the average of the log of real wages of heads of household with a college degree or more and the average of the log of the real wages of heads of household that have a high school degree over a sample of wage workers. See appendix A.1.2 for additional details on the calculation of the supply of college graduates and the college premium.

0.26, which is the fraction of heads of household with a college degree in the CPS sample in 1985. Conditional on these three fixed values, I choose the rest of the parameters to match:

- a skill premium of 39%, which is the value of the college premium in 1985, as shown in figure 1.8,
- a labor share of output of 63%, which is the average labor share of non-farm business sector output between 1980 and 1985 calculated by the Bureau of Labor Statistics,
- a ratio of liabilities plus equity in the non-financial sector to non-financial private sector output of 0.88,[29]

[29]The ratio of liabilities plus equity of the non-financial sector is the sum of the total liabilities of the non-financial non-corporate sector as reported by the Federal Reserve Bank of St. Louis Economic Data (FRED) – time series NNBTILQ027S – plus total liabilities plus equity of the non-financial corporate sector – time series NCBLEYQ027S – divided by the total nominal output of the private non financial sector as reported by the BEA. See appendix A.1.3 for additional details.

- a population share of entrepreneurs of 7.80%, which is the fraction of entrepreneurs calculated from the PSID for 1985,
- a population share of entrepreneurs that are high skill of 4.2%, which is the fraction of entrepreneurs with a college degree or more calculated from the PSID for 1985,
- a share of households transitioning from being wage workers into entrepreneurship of 2.4%, which is the fraction of wage workers transitioning to entrepreneurship calculated from the PSID for 1985,
- and a share of new entrepreneurs of 23%, which is the fraction of households that switched to entrepreneurship in 1985 over the total number of entrepreneurs in 1985.

Table 1.3 reports the parameters calibrated jointly with the equilibrium of the model and table 1.4 shows the calibration targets and the model generated moments. The model matches well the population share of entrepreneurs, the population share of entrepreneurs that are high skill, and the skill premium, all of which are key for my quantitative exercise. However, the share of new entrepreneurs and the transition rate of workers into entrepreneurship implied by the model are larger than the corresponding values calculated from the data. The model also generates substantial wealth inequality although the calibration did not intend to match any of the moments of the wealth distribution. As reported by Bricker, Henriques, Krimmel, and Sabelhaus (2016), the Gini coefficient on wealth in 1989 was 0.84, while the Gini coefficient implied by the model for the same year is 0.83. Similarly, the share of wealth accrued by the top 1% of the population was 32% in 1989 and 34.2% in the model.[30]

1.4 Results

1.4.1 Transition

This section shows the main quantitative results of my model. The economy is assumed to be at a stationary equilibrium in 1985. Then, individuals learn about the future path of the

[30] The moments reported by Bricker et al. (2016) are start in 1989. Hence, the model moments are taken from the fourth year of the transition generated by the model, which corresponds to 1989. For 1985, the model generates a Gini coefficient of 0.84 and a share of wealth of the top 1% of 33.2%.

Table 1.3: Calibrated Parameters in the Benchmark Model

Parameter		Value
Capital Share	τ	0.48
Labor Share	ψ	0.50
Borrowing Limit	λ	2.5
Entry Cost	κ	0.10
Relative Productivity of Low Skill	θ_L	0.96
Autocorrelation of z	ρ_z	0.82
Standard Dev. of ϵ_z	$\sigma_{\epsilon,z}$	0.22

Note: Table 1.3 reports the set of calibrated parameters and their corresponding values. The data and analogous model generated moments are reported in table 1.4.

Table 1.4: Data and Model-Generated Moments

	Data	Model	Source
log-Skill Premium	0.39	0.39	CPS
Labor share	0.63	0.63	BLS
Debt and Equity to GDP	0.88	0.90	Flow of Funds
Share of Entrepreneurs (%)	7.80	7.87	PSID
Share High Skill of Entrepreneurs (%)	4.20	4.15	PSID
Transition Rate (%)	2.40	3.10	PSID
Share of New Entrepreneurs (%)	23.1	35.0	PSID

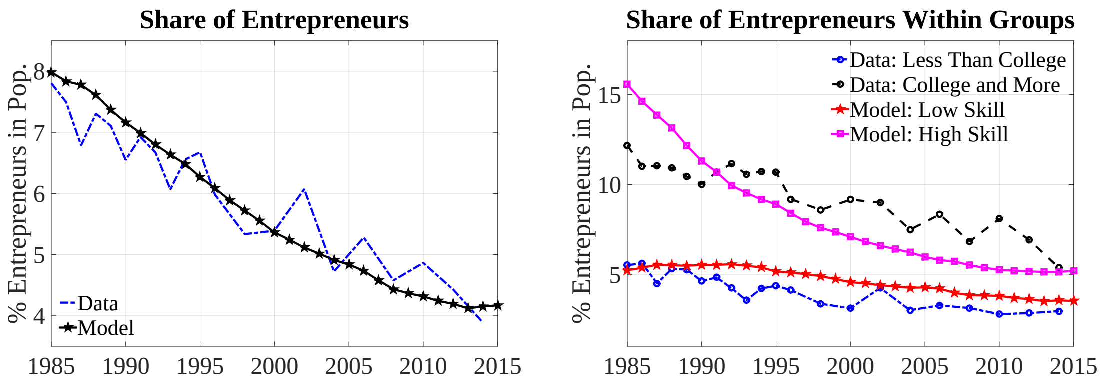
Note: Table 1.4 shows the set of moments that are targeted to choose the parameters in table 1.3.

three aggregate time series (the cost of capital goods, the share of skilled workers, and the relative productivity of high skill workers), the evolution of the distribution of individuals over idiosyncratic states, and consequently, factor prices. In other words, individuals have perfect foresight about the evolution all the relevant variables from 1985 to the infinite future.[31] In my benchmark exercise, the time series of each of these variables are as shown in figure 1.8 and remain constant after 2015 for the rest of the transition.[32] The left panel of figure 1.9 shows the evolution of the fraction of entrepreneurs in the data (blue-dashed line) and in the model (black-starred line). The model accounts well for the decline in the share of entrepreneurs in the population and the speed of the decline. In the model, the

[31]I choose 1985 as my starting point because of data availability. However, as I show in the robustness section, my main quantitative results do not change substantially if I assume that the economy was at steady state in 1970.

[32]The entire transition consists of 300 periods, from 1985 to 2285.

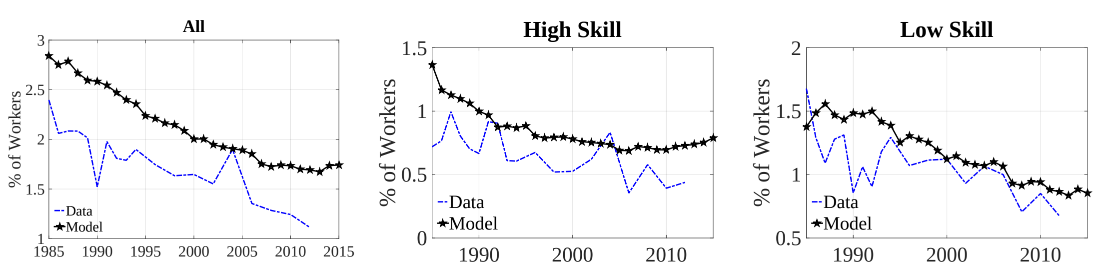
Figure 1.9: Evolution of the Share of Entrepreneurs



Note: The left panel of figure 1.9 shows the evolution of the share of entrepreneurs as calculated from the PSID data (blue dashed line) and the share of entrepreneurs implied by the model (black-starred line). The right panel shows the share of entrepreneurs within skill groups.

fraction of entrepreneurs drops 3.8 percentage points between 1985 and 2014, accounting for almost all of the 3.9 percentage points decline observed in the data. The model is also consistent with differences in the evolution of the share of entrepreneurs between high and low skill workers, as shown in the right panel of figure 1.9. In the model, however, the decline in the share of high skill entrepreneurs is faster than the decline observed in the data (for the group of college graduates), especially in the first years of the transition. The model more closely matches the evolution of the share of low skill workers, although it underpredicts the speed of the decline. Finally, the decline in the entry rate implied by my model economy is similar to the decline observed in the data, as is shown in figure 1.10.

Figure 1.10: Transition Rate of Workers to Entrepreneurship



Note: Figure 1.10 shows the evolution of the transition rate into entrepreneurship as calculated from the PSID data (blue-dashed line) and the corresponding measure obtained from the model. The transition rate is equal to the proportion of households that were not entrepreneurs in period t but transitioned to entrepreneurship in period $t+2$.

1.4.2 Decomposition

What is the relative contribution of each of the exogenous trends to the decline in the share of entrepreneurs? To answer this question, I study the evolution of the fraction of entrepreneurs considering the effect of each of the aggregate trends, starting with the skill-biased technological progress, $A_{H,t}$.[33] Here I consider the same values of $A_{H,t}$ used in my baseline exercise but I fix the values of $p_{k,t}$ and H_t to their levels in 1985 (the initial stationary equilibrium). The evolution of the population share of entrepreneurs in this case is shown in the left panel of figure 1.11. The black-starred line is the population share of entrepreneurs implied by the model in the baseline case and the red-circled line shows the proportion of entrepreneurs for the case in which only $A_{H,t}$ moves. This can be thought of as the direct effect of skill-biased technological change on the share of entrepreneurs. In this case, the proportion of entrepreneurs drops 2.1 percentage points between 1985 and 2015, or 55% of the overall decline implied by the model (and 53% of the decline in the data). The discrepancy between my baseline results and this case is explained by the response of the low skill individuals. First, the center panel of figure 1.11 shows that an increase in $A_{H,t}$ reduces the share of high skill entrepreneurs slightly more than in the baseline case. This is because an increase in $A_{H,t}$, coupled with the relative scarcity of high skill workers in the economy, makes the wages for this group increase very fast, decreasing the incentives for high skill workers to become entrepreneurs. On the other hand, although low skill workers are relatively less able as entrepreneurs, they experience an increase in their profits as the high skill workers that they hire are, in fact, more productive. Moreover, the relative abundance of low skill workers implies that their wages do not increase as much compared to my baseline results. Consequently, the share of low skill entrepreneurs increases, as shown by the red-circled line in the right panel of figure 1.11.

Next, I consider the transition of the economy when both $A_{H,t}$ and H_t change over time. In this case, the share of entrepreneurs declines even further, as shown by the green-squared line in the left panel of figure 1.11. An increase in the relative supply of high skill workers has two opposite effects. First, it depresses the wage of high skill workers, increasing the incentives for this group to become entrepreneurs. Moreover, since high skill individuals

[33] This decomposition might be influenced by the order in which each exogenous trend is considered. I have run several robustness checks for different combinations, yielding similar results.

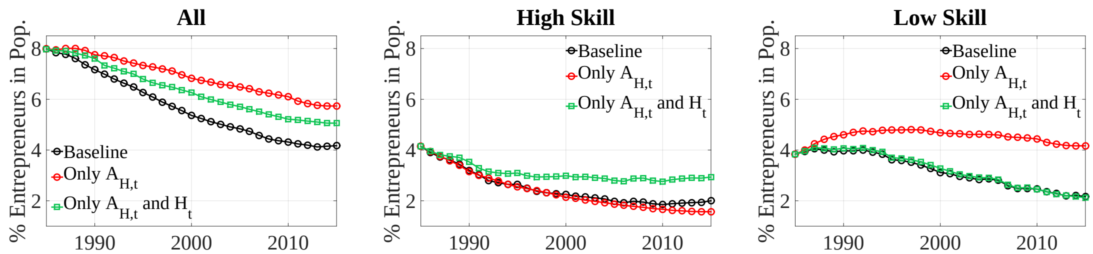
are more productive than low skill individuals as entrepreneurs (recall the differences in θ_s), the total effect is an increase in the share of high skill entrepreneurs in the economy, as shown by the green-squared line in the center panel of figure 1.11. On the other hand, the surge in the demand for labor and the decreasing share of low skill workers push their wages up, decreasing the incentives for low skill workers to become entrepreneurs. Then, because low skill workers still represent the largest share of the population, the overall proportion of entrepreneurs declines. Taken together, the increase in the productivity of high skill workers and the increase in the supply of high skill labor account for 75% of the drop in entrepreneurship implied by the model.

Finally, the decline in the relative price of capital brings the green-squared line in the left plot of figure 1.11 to the black-starred line. Because of the complementarity between capital and high skill workers, a decrease in the price of capital goods raises the demand for high skill workers, increasing their wages and depressing their incentives to become entrepreneurs. This brings down the share of high skilled individuals that decide to run their own firm while holding steady the share of low skill entrepreneurs in the economy.[34]

I find similar results when looking at the transition rate into entrepreneurship and the exit rate of entrepreneurs. Appendix figure A.13 displays the time series of the share of workers that start a new firm in the next period under the same decomposition that I consider in this section. Figure A.14 shows the corresponding time series for the exit rate of entrepreneurs. Similarly the results on the population share of entrepreneurs, the skill-biased technological change explains about 50% of the decline in the transition rates in and out of entrepreneurship generated by the model. In summary, each of the three trends considered in my quantitative exercise explains an important fraction of the decline in entrepreneurship and implies different responses of high and low skill individuals along the transition. The direct effect of skill-biased technological change explains the lion's share of the overall decline and explains half of the drop in the proportion of entrepreneurs and the entry and exit rate dynamics generated by the model.

[34] Varying the order of the shocks in this decomposition does not alter the proportion of the decline in the share of entrepreneurs explained by each of the exogenous trends that I consider. For instance, appendix figure A.12 shows a decomposition in which I first let $A_{H,t}$ vary over time, then $p_{k,t}$, and finally, H_t. In such a case, the first two exogenous processes explain three-quarters of the total decline in the share of entrepreneurs implied by the model.

Figure 1.11: DECOMPOSITION OF THE SHARE OF ENTREPRENEURS



Note: Figure 1.11 shows the time series of the population share of entrepreneurs implied by the model. The black-starred line shows the share for the baseline case, the red-circled line considers only the evolution of $A_{H,t}$, while the green-squared line considers $A_{H,t}$ and H_t. The center and right panel show similar statistics for high and low skill workers.

1.4.3 Wages, Skill Premium, and Productivity

In the model, individuals decide in each period whether to run a firm or work for a wage, comparing the utility value of each of these options. Intuitively, an increase in wages will reduce the incentives to start a firm, more so for those workers whose wage grows faster. Since both the decrease in the price of capital goods and the increase in the productivity of high skill workers increase the marginal productivity of labor, the productivity threshold that makes individuals indifferent between working for someone else or running a firm also increases, shrinking the share of entrepreneurs in the economy. The top left panel of figure 1.12 shows the log-level of wages of high and low skill workers generated by the model. Both are increasing, reducing the share of high and low skill entrepreneurs. However, since the wages of high skill workers rise faster, generating the increase in the skill premium displayed in the top right panel of figure 1.12, the share of entrepreneurs declines more within the group of high skill workers. Importantly, only entrepreneurs with high managerial abilities remain active, and the average value of z rises, as shown in the bottom left panel of figure 1.12. The profits of the remaining entrepreneurs also increase because the workers they hire are more productive.

In my model, the excess return to entrepreneurship, that is, the average profits of the active entrepreneurs relative to the profits they would obtain as workers, also increases. To see this, the bottom right panel of figure 1.12 shows the percentage increase in the average excess returns with respect to the value in 1985 for high and low skill individuals. Relative to the initial steady state, the excess return increases by 15% for the group of high skill entrepreneurs. This is qualitatively consistent with the evidence presented by Michelacci and Schivardi (2016) that reports a large increase in the excess return, particularly for entrepreneurs with postgraduate studies. Low skill individuals are, on average, less productive, and the excess return decreases relative to the initial steady state as the wages of low skill workers increase during the transition.[35]

Similar to the evidence shown in figures 1.4 and 1.5, my model predicts, first, an increase in the mean labor earnings of individuals that switch from worker to entrepreneur, and second, a stronger increase within high skill individuals. This is shown in figure 1.13 which displays the average labor earnings of high and low skill individuals before transitioning into entrepreneurship (black-starred line) and the income of those individuals that stayed as workers. My model results are qualitatively consistent with the large increase in labor earnings for individuals that switch from workers to entrepreneurs.

1.4.4 Robustness

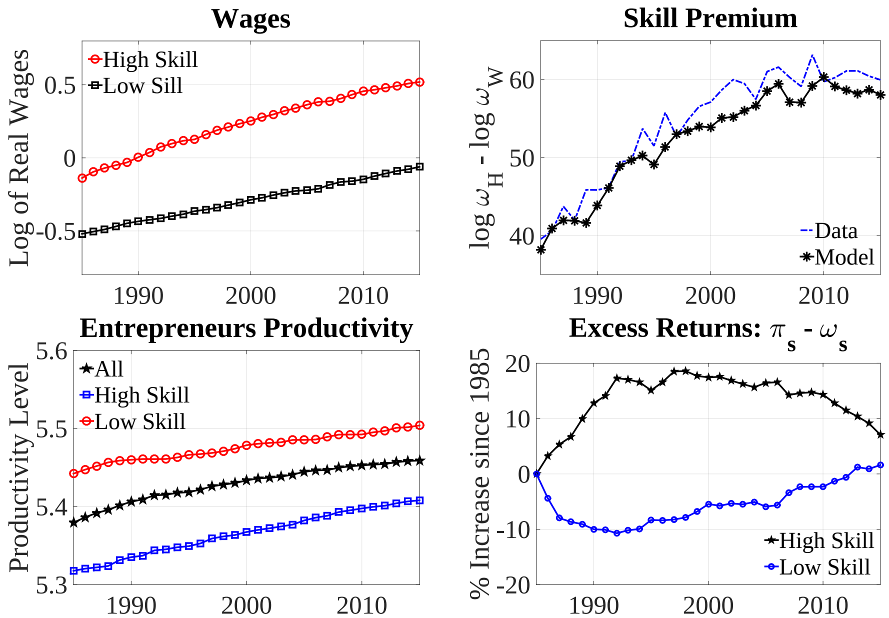
Firm Creation Cost and Borrowing Constraints

A possible explanation for the decrease in new business formation is the increasing cost of regulation in the United States, which affects existing and potential entrepreneurs.[36] Arguably, an increase in the regulatory burden would deter firm creation in a similar way to the deterrence effect of an increase in the value of κ, the creation cost in the problem of the entrepreneurs. Hence, in this section I ask, what is the level of κ that generates the same proportion of entrepreneurs in 2014 conditional on the parameter values of 1985? I find that the value of κ required to reduce the proportion of entrepreneurs to the level observed in 2014 is almost seven times the value of κ in my calibration exercise. Denote this new level of the entry cost by $\tilde{\kappa}$. Next, I study the transition observed in the model generated by a linearly increasing trend of the entry cost. As in my previous exercise, I

[35] Michelacci and Schivardi (2016) define the excess return as the difference between the average entrepreneurs' returns and average labor earnings. In their analysis, entrepreneurs' returns are the sum of labor earnings of self-employment, dividends, and expected capital gains. My model only considers the dividends part of entrepreneurs' returns.

[36] The cost of regulation is difficult to measure. One proxy for this cost is the number of restrictions in the administrative code. Al-Ubaydli and McLaughlin (2017) provide an estimate of the number of restrictions and words that indicate that a specific action is prohibited or required for firms. Using this dataset, I find that the average number of restrictions within 2-digit NAICS industries grew 75% from 1985 to 2015.

Figure 1.12: Wages, Skill Premium, and Productivity

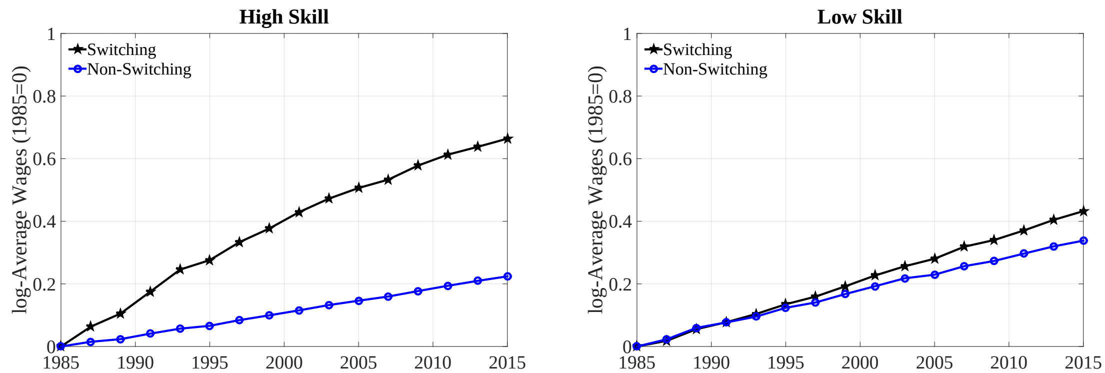


Note: The top-left panel of figure 1.12 shows the evolution of the (log) wages in the model. The top right panel shows the college premium in the data and the skill premium implied by the model. The bottom left panel shows the evolution of the (log) average skill level of active entrepreneurs calculated as $\overline{z}_t = \sum_{s \in \{H,S\}} \int z_{i,t} d^s_{i,t} d\mu^s_t$, whereas the bottom right panel shows the entrepreneurial excess return normalized 1985.

assume that the economy is at a stationary equilibrium in 1985. Then, the agents learn the entire sequence of κ_t. I assume that κ_t increases between 1985 and 2015 and remains fixed at the value in 2015 for the rest of the transition. For the initial value of κ_t, I take the level used in my baseline calibration, and for the terminal level, I choose $\tilde{\kappa}$.

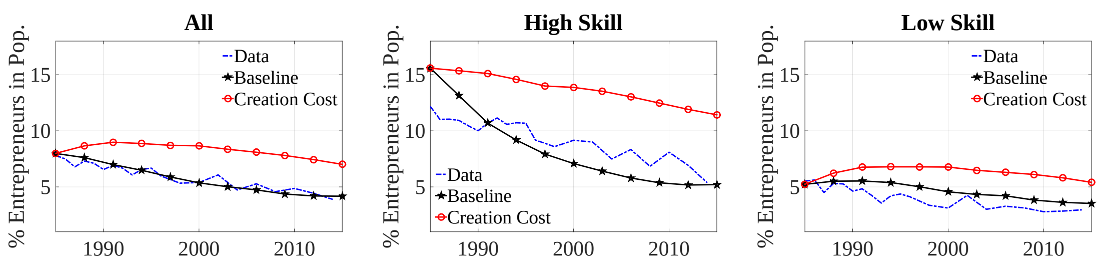
The left panel of figure 1.14 shows the evolution of the population share of entrepreneurs resulting from an increase in the cost of firm creation. To facilitate the comparison with my previous results, the figure also displays the evolution of the population share of entrepreneurs for my baseline calibration and the population share of entrepreneurs in the data. First, notice that the decline in the share of entrepreneurs implied by an increase in κ_t is more moderate than the decline observed in the data. This is because, in anticipation of higher future costs, some households decide to transition earlier into entrepreneurship, raising the share of entrepreneurs above the original stationary level. The increase in the share of entrepreneurs is mostly explained by the response of low skill individuals as shown

Figure 1.13: Labor Earnings for Switching and Non-Switching Workers



Note: Figure 1.13 shows the average labor earnings of individuals *prior* switching to entrepreneurship and the average labor earnings of those individuals that stayed as workers.

Figure 1.14: Effects of an Increase in κ



Note: The left panel of figure 1.14 shows the share of entrepreneurs implied by a linear increase in the cost of creating a firm (κ). The center and right panel show same statistics within the group of low and high skill households.

in the right panel of figure 1.14. Finally, the center panel displays the evolution of the share of high skill entrepreneurs, which is flatter than the evolution implied by my baseline exercise. In summary, an increasing cost of regulation, in the form of an increase in the cost of firm creation, generates a small decline in the share of entrepreneurs along the transition and does not seem to account for the rapid decrease in the share of entrepreneurs among high skill individuals.[37]

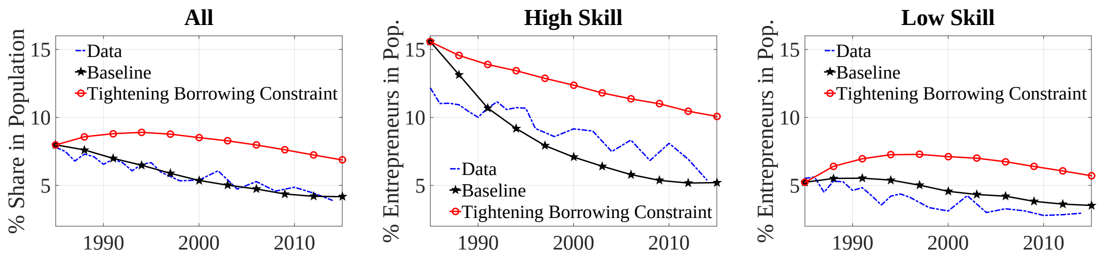
[37]These results might overestimate the real effect of the increase in the entry cost in my model. If one takes the entry cost as a proxy for the cost of regulation imposed by the tax code and other mandatory rules that firms need to follow, then the entry cost must have increased only 75% based on my calculations using the data collected by Al-Ubaydli and McLaughlin (2017). In such a case, keeping the rest of the parameters as in 1985, my model would predict a decrease of 0.5 percentage point in the share of entrepreneurs, much less than the drop of 3.9 percentage points observed in the data.

A tightening of the borrowing constraint could also generate a decrease in the share of entrepreneurs. There are at least two possible problems for such channel to be quantitatively important. First, given the large increase of house prices observed before the Great Recession, it is unlikely that the borrowing constraints have tightened over time, as many entrepreneurs use their house as a collateral to start a firm. Moreover, if the borrowing constraints have become tighter, then one would expect that the rate at which households and entrepreneurs get rejected by a financial institution when asking for a loan to increase. However, data from the SCF shows that the rate of rejection has decreased, if anything, across the population in general, and for entrepreneurs in particular.[38] Second, in the context of my model, a tightening of the borrowing constraint is inconsistent with the decrease in the share of low skill entrepreneurs observed in the data. In fact, the model predicts an increase in the share of entrepreneurs within this group. Figure 1.15 shows the time series of the share of entrepreneurs under the assumption that the value of λ decreases by a third between 1985 and 2015, keeping fixed the rest of the parameters as in 1985. In this case, the population share of entrepreneurs declines by half, mostly because of the decrease in the share of high skill entrepreneurs. However, within the group of low skill individuals, the share of entrepreneurs increases over time. This differential response comes from the differences in managerial ability of high and low skill individuals. Because high skill individuals are relatively more productive as entrepreneurs they run larger projects that require more resources. Therefore, a tightening of the borrowing constraint affects them the most, whereas the effect for low skill individuals is weaker because they are less able as managers and run smaller projects.[39]

[38]The share of households whose head is between 22 and 60 years old that asked for a credit but where rejected has remained constant around 25% since 1989. For entrepreneurs, the rate of rejection declined from 25% in 1992 to less than 15% in 2016. Separating the sample in different age groups or considering heads of households more than 60 year old does not change this result. Alternatively, it is possible that individuals do not get rejected because they never asked for a credit in the first place. The SCF provides information on the share of households (entrepreneurs) that did not ask for a credit because they expected to be reject by the financial institution. Similarly to the rejection rate, the share of households that did not ask for a credit because they expected a rejection has remained constant around 15% (12% within entrepreneurs) between 1989 to 2007, increased during the Great Recession, and then declined again in 2016.

[39]Importantly, these results do not depend on the specific form of the borrowing constraint. The results showed in appendix A.3.1 consider a more standard borrowing constraint of the form $p_k k \leq \lambda a$. I find a similar decline in the share of entrepreneurs despite the fact that the borrowing constraint is becoming more relaxed over time as the cost of capital declines.

Figure 1.15: Effects of a Decrease in λ



Note: The left panel of figure 1.15 shows the share of entrepreneurs implied by a linear decrease in the value of λ. The center and right panel show same statistics within the group of low and high skill households.

Myopic Transition

So far I have assumed that agents have perfect foresight about the future path of the aggregate state of the economy. Alternatively, one could consider that agents are surprised every period about changes in the aggregate variables and expect that the current state of the economy remains fixed for the infinite future. In this sense, agents are perfectly myopic. This assumption does not change the equilibrium definition, but it does modify the information set available for the agents. Hence, one needs to change the solution algorithm accordingly. Appendix A.2 explains in detail the algorithm used to solve the transition path in this case. The upper left panel of figure 1.16 shows the evolution of the fraction of entrepreneurs in this case, along with the time series for the perfect foresight case and the share of entrepreneurs calculated from the data. Overall, the evolution of the share of entrepreneurs is quite similar, both qualitatively and quantitatively. Relative to the perfect foresight case, the overall fall in the share of entrepreneurs and the speed of decline remain basically unchanged if one assumes that entrepreneurs learn every period about the current aggregate state of the economy. Therefore, I conclude that assuming perfectly myopic agents does not affect the results of my model.

Smooth Transition of Aggregates

How would my results change under an alternative assumption about the evolution of the aggregates after 2015? To answer this question, I assume that the price of capital goods, the relative productivity of high skill workers, and the supply of high skill workers all keep a constant growth rate equal to the average growth rate observed during the period 2005-2015. I assume the growth rate of each trend declines geometrically to reach 0 growth in 2035. The upper right panel of figure 1.16 shows that the share of entrepreneurs and the decline implied by the model remain basically the same as in my baseline results.

General Equilibrium

In the benchmark results, I have assumed a fixed interest rate. This was with the explicit purpose of highlighting the effects of the change in wages on the decision to become an entrepreneur while muting the general equilibrium effect of the change in the interest rate on the production cost and individuals' savings. In this section, I study how my results change if I solve for the equilibrium interest rate along with the wages of high and low skill workers.[40] The bottom left panel of figure 1.16 shows the evolution of the share of entrepreneurs in the data, in the benchmark small open economy case and in the general equilibrium case.[41] In the latter, the decline in the share of entrepreneurs is half of the decline generated by my benchmark case. This is because the interest rate increases substantially along the transition path, increasing the rate of households's saving. The increase in individuals' savings has two effects on entrepreneurial profits. First, more savings imply more capital for the firms, increasing the profits of entrepreneurs. Second, more wealth relaxes the borrowing constraint, allowing entrepreneurs to operate firms closer to their optimal size. These two effects increase the value of being an entrepreneur. Consequently, the general equilibrium feedback dampens the effects of the changes in the wages on the share of entrepreneurs. Overall, the general equilibrium version of the model predicts a decline of 2.0 percentage points in the share of entrepreneurs, which is about 52% of the decline generated in the fixed interest rate case. Consequently, although considering general equilibrium changes my results quantitatively, the changes in wages and profits induced by the three aggregate trends I consider can account for at least half of the fall in the share of entrepreneurs observed in the data.

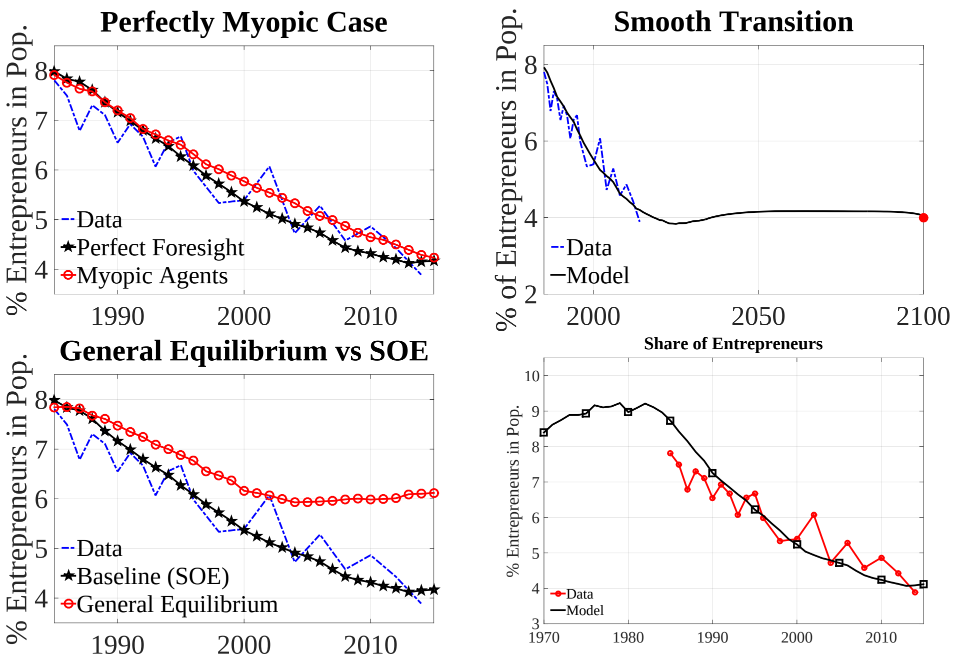
[40]Since this is now a closed economy, one needs to pin down a particular value of capital-to-output ratio at the initial steady state. I do that by choosing value of β to that the capital-to-output ratio is equal to three in the stationary equilibrium at the beginning of my simulation. The corresponding value of β is 0.88, which is what I use in my baseline calibration.

[41]Appendix figure A.11 shows the evolution of the skill premium and the interest rate implied by the model.

A Longer Transition

In my baseline exercise, I have assumed that the economy is in steady state in 1985. Then agents learn about the path of aggregate variables from that year to the infinite future. However, the relative price of investment has shown a declining trend starting several years before 1985, so one might wonder how much my quantitative results depend on the assumption that the economy is at steady state in 1985. To address this concern, here I assume that the economy is at its stationary equilibrium in 1970, and then individuals observe the evolution of the price of capital, the evolution of the productivity of high skill workers, and the supply of high skill labor. As in my previous results, here I take the values of $p_{k,t}$ and H_t directly from the data, and I change the value of $A_{H,t}$ to reproduce the skill premium observed in 1970. Then, I assume that each exogenous process remains constant after 2015. The bottom right panel of figure 1.16 shows the evolution of the share of entrepreneurs in the data and the model. First, notice that starting the economy in 1970 does not affect the ability of the model to closely follow the decline in the share of entrepreneurs from 1985 on. Second, the model predicts that the share of entrepreneurs was more or less stable between 1970 and the early 1980s, and started its decline in the mid-1980s. The data necessary to construct my preferred definition of entrepreneurs start in 1984. However, one can look at the evolution of the share of self-employed business owners to get a sense of the evolution of the share of entrepreneurs before 1985. This is show by the squared-red line in the bottom right plot of figure 1.16. Between 1970 and 1985, the college premium decreased and then bounced back, explaining why my model predicts a flat rate of entrepreneurs in the first years of the transition. Therefore, I conclude that a longer transition starting from 1970 does not substantially change my baseline, post-1985, results. However, the model cannot account for the increasing trend of entrepreneurs in the early 1980s. Some other factors (such as changes in taxation or changes in the age distribution of the labor force) might explain the increase in the share of entrepreneurs in the late 1970s and early 1980s.

Figure 1.16: Share of Entrepreneurs for Different Robustness Exercises



Note: The upper left panel of figure 1.16 compares the evolution of the share of entrepreneurs in the baseline perfect foresight case with the share of entrepreneurs in the perfectly myopic case. The upper right panel shows the transition path of the share of entrepreneurs under the assumption that aggregate variables grow at a decreasing rate after 2015. The bottom left panel of figure 1.16 compares the baseline, fixed interest rate case with the evolution of the share of entrepreneurs in the general equilibrium case. Finally, the bottom right panel displays the share of entrepreneurs implied by the model under the assumption that the economy was in steady state in 1970.

1.5 Policy Analysis

Several researchers have considered the decline in firm creation to be a negative outcome that should be addressed by policy. In my model, a decline in firm creation is equivalent to a fall in the transition rate of workers into entrepreneurship. In this section I study the aggregate response of the economy to a subsidy that intends to increase the rate of entry of new entrepreneurs to its 1985 level conditional on the parameter values in 2015. Consider now a policy reform that directly relaxes the borrowing constraint of the entrepreneurs. Specifically, I assume that the government gives a subsidy of ι_t to finance the cost of inputs to every entrepreneur in the economy. Hence, entrepreneurs face a new collateral constraint given by

$$(1-\iota_t)\left(p_{k,t}k_t+\omega_{H,t}\left(\Theta_t,\mu_t\right)n_{H,t}+\omega_{L,t}\left(\Theta_t,\mu_t\right)n_{L,t}\right)\leq\lambda a_t.$$

Although this subsidy affects every entrepreneur, it has a larger impact on small and new entrepreneurs, who typically have less wealth than large, established entrepreneurs. I assume that the government collects revenues using an equal linear tax on workers' and entrepreneurs' income. Denote this tax by τ_g. Then, the government budget constraint is given by

$$\sum_{s\in H,L}\int\tau_g\left(d_t^s\left(\Omega_t,p_{k,t},\mu_t\right)y_t\omega_{s,t}+d_t^s\left(\Omega_t,p_{k,t},\mu_t\right)\pi_{s.t}\left(a_t,z_t\right)\right)\mu_t^s\left(\Omega_t\right)=$$
$$\sum_{s\in H,L}\int d_t^s\left(\Omega_t,p_{k,t},\mu_t\right)\left(\left(p_{k,t}k_t^s+\omega_{H,t}\left(\Theta_t,\mu_t\right)n_{H,t}^s+\omega_{L,t}\left(\Theta_t,\mu_t\right)n_{L,t}^s\right)\right)\iota_t d\mu_t^s\left(\Omega_t\right). \quad (1.10)$$

The equilibrium definition for this case is similar to the definition in section 1.3.2, with the additional condition that the government must balance the budget in every period. Using this simple policy, I run the following experiment. I start as if the economy is in a stationary equilibrium conditional on the parameter values of 2015, and I compare this economy to a new stationary economy in which the subsidy is such that the entry rate of new entrepreneurs is equal to 2.4%, which is the transition rate from workers into entrepreneurship observed in 1985. Columns (1) and (2) of table 1.5 compare aggregate output, total factor productivity (TFP), the tax burden as a percentage of output, and other aggregates at the stationary equilibrium of both economies.[42] To reach an entry rate of entrepreneurs as in 1985, the government imposes a tax that generate revenues that are equivalent to 3.16% of aggregate output. The increase in the entry rate induces a rise in the fraction of entrepreneurs in the population of 2.42 percentage points and an increase in aggregate output of 4.0%. TFP increases 9.20% in the new steady state. This happens for two reasons. First, on the extensive margin, some production factors are reallocated from the non-entrepreneurial sector to the new entrepreneurs that are, on

[42] TFP is defined as $Y\left(K^{0.33}L^{0.67}\right)^{-1}$. Here, Y is aggregate output, K is the sum of the capital utilized by all the entrepreneurs and the non-entrepreneurial sector, and L is the size of the labor force, which is normalized to 1.

average, more productive than the firms in the non-entrepreneurial sector. Second, on the intensive margin, already existing entrepreneurs can run firms closer to their optimal, unrestricted, scale.

The subsidy also induces higher welfare, measured in consumption equivalents. In particular, the average consumption equivalent required to make individuals indifferent between the baseline steady state and the steady state with a subsidy to firms is 0.04; that is, individuals are willing to give up some consumption to live in an economy where the subsidy is in place.[43] The welfare gains, however, are not distributed equally across the population. High skill individuals experience a larger increase in welfare because they are able to run larger firms. Low skill workers also enjoy an increase in welfare as their wages go up because of the increase in labor demand. Low skill entrepreneurs experience the lowest increase in welfare. This is because, although they receive a subsidy that relaxes their borrowing constraint, the tax is large and pushes down their consumption, and hence welfare, although not enough to reduce it below the level of the unsubsidized economy.

In fact, the subsidy is much more effective in inducing high skill workers to become entrepreneurs than it is for low skill workers. The fraction of high skill individuals that are entrepreneurs increases 1.56 percentage point, from 2.41% to 3.96%, whereas the fraction of low skill entrepreneurs rises 0.88 percentage points from 1.26% to 2.14%. Alternatively, the government could impose a tax that maximizes the welfare of the economy. If the government considers an equally weighted average of utility across the individuals in the economy, the maximum level of utility is reached with a subsidy of 2.58% of GDP, as column (4) in table 1.5 shows. Output, productivity, and the share of entrepreneurs also increase in this case. I conclude that a policy that aims to subsidize firms through a credit line that relaxes the borrowing constraint with the goal of increasing the entry rate of entrepreneurs to its level in 1985 would generate substantial benefits to the economy.

The previous exercise shows that the decline of entrepreneurship can be alleviated by a subsidy to entrepreneurs' cost. What would the transition of the economy looked like is such policy had been implemented in 1985? Figure 1.17 shows the time series of the share

[43]The average consumption equivalent is the equally weighted average of the value $\omega(\Omega)$ that solves the equation $(1+\omega_s(\Omega))^{1-\sigma} V^*_{s,t}(\Omega) = \tilde{V}_{s,t}(\Omega)$ where $V^*_{s,t}(\Omega)$ is the value of an individual in the original stationary equilibrium without subsidies and $\tilde{V}_{s,t}(\Omega)$ is the value at the new stationary equilibrium, both conditional on the same idiosyncratic states, Ω.

Table 1.5: STEADY STATE COMPARISON OF A SUBSIDY TO FIRM COSTS

	(1)	(2)	(3)	(4)	(5)
	Baseline	1985-level		Optimal	
		Level	Δ	Level	Δ
Subsidy (% of GDP)	-	3.16		2.58	
Entry Rate (%)	1.55	2.40		2.35	0.80
Aggregate Output	1.50	1.56	4.0%	1.56	4.0%
Total Factor Productivity	0.87	0.95	9.2%	0.94	8.0%
Entrepreneurs All (%)	3.68	6.10	2.42	5.85	2.17
High Skill (%)	2.41	3.96	1.55	3.77	1.36
Low Skill (%)	1.26	2.14	0.88	2.08	1.08
Entrepreneurs W/High Skill (%)	6.34	10.30		9.80	
Entrepreneurs W/Low Skill (%)	2.03	3.48		3.38	

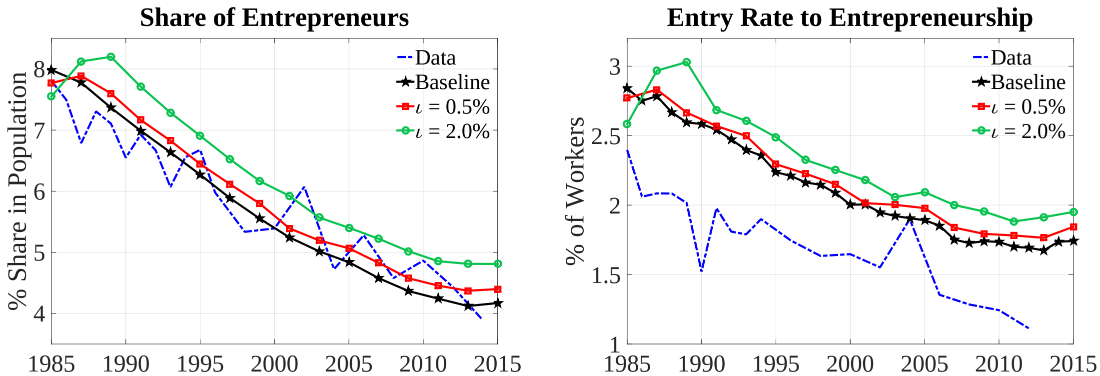
Note: Table 1.5 compares the macroeconomic aggregates of different stationary economies. Column (1) shows the results for the baseline stationary economy if the parameters were as in 2015 and remain constant for the infinite future. Column (2) uses the same set of parameters but introduces a subsidy to finance the production costs of the entrepreneurs financed with linear tax for all individuals in the economy. The subsidy level is $\iota_t = 6.5\%$ of the total cost of production. Column (4) shows the same statistics under the assumption that the government implements the subsidy that maximizes the average welfare of the stationary economy given the parameter values in 2015. The value of the subsidy is $\iota_t = 5.73\%$.

of entrepreneurs both in the data, for my baseline results, and for different levels of the subsidy. As in the steady state comparison, a positive subsidy generates an increase in the level of the share of entrepreneurs in each year of the transition, however, it does not change the slope of the time series. In other words, although a fixed subsidy is effective in generating a larger share of entrepreneurs, it is unable to undo the impacts of the technological changes affecting the economy that have led to an equilibrium decline in the share of entrepreneurs and the entry rate to entrepreneurship.

1.6 Conclusion

In this paper, I have studied the causes of the decline in the share of entrepreneurs in the US population, and I have linked such decline to technological changes experienced by the US economy in recent decades. In doing that, I have argued that the same technological changes that have given rise to the increase of returns to high skill workers are responsible for the decline in the share of entrepreneurs observed in the United States over the last

Figure 1.17: Entrepreneurs and Transition for Different Subsidies



Note: The left panel of figure 1.17 shows the time series of the share of entrepreneurs in the data for four different levels of input subsidies to entrepreneurs. The right panel shows the transition rates into entrepreneurship.

three decades.

I provide new evidence on the fall in the share of households participating in entrepreneurial activities and on the share of households transitioning into entrepreneurship. Moreover, I show that the decline in the proportion of entrepreneurs has been concentrated among individuals with high levels of educational attainment. Additional empirical evidence suggests that the skill level of new entrepreneurs has increased over time, which is consistent with an increase in the selection of individuals with higher managerial abilities.

Building on this evidence, I study an entrepreneurial choice model where the increase in the wage of high skill workers is the equilibrium outcome of the interplay of three exogenous trends, namely, the decrease in the price of capital goods, the increase in the supply of skilled labor, and the increase in the productivity of high skill workers. My model is able to account for the level and the speed of the decline in the share of entrepreneurs. In my baseline exercise, the model generates a decline of 3.8 percentage points in the share of entrepreneurs, which is almost all of the decline of 3.9 percentage points experienced in the United States over the last 30 years. My model is also consistent with the differential decrease in the share of entrepreneurs among high and low skill individuals and the observed decline in the transition rate into entrepreneurship. Moreover, I find that each of the exogenous trends is quantitatively important in explaining the drop in the fraction of entrepreneurs in the economy. The increase in the productivity of high skill workers accounts for half of the decrease in the share of entrepreneurs, whereas the other half is equally explained by the decrease in the price of capital goods and by the increase in the supply of high skill workers.

Taken together, my results indicate that a large fraction of the decline in the pace of firm creation and entrepreneurship is the result of the same technological improvements that have increased the returns to high skill workers over the last 30 years. Such changes have naturally led to a reduction in entrepreneurship and entrepreneurial dynamism. However, viewed trough the lens of my model, the decline in entrepreneurship, firm creation, and overall dynamism of the US economy should not be cause for concern. This does not imply, there is no role for government intervention. To the extent that the government can ease the borrowing constraint faced by entrepreneurs–the only source of inefficiencies in my model–the share of entrepreneurs would increase, as well as output and productivity.

Chapter 2

Skewed Business Cycles

2.1 Introduction

This paper studies the cyclicality of the distribution of the growth rate of firm-level outcomes.[1] In the prior literature, recessions have been characterized as a combination of a negative first-moment (mean) shock and a positive second-moment (uncertainty) shock (Bloom, 2014). In this paper we argue that recessions are also accompanied by negative third-moment (skewness) shocks implying that, during economic downturns, a subset of firms does extremely badly, leading to a left tail of large negative outcomes. Consequently, the skewness of the growth rates is procyclical.

Using Census firm-level panel data for the United States and firm-level panel data of publicly traded firms for the United States and for more than thirty five other countries, we show that the cross-sectional skewness of the distribution of several firm-level outcomes, such as sales growth, employment growth, and stock returns, is strongly procyclical, declining sharply during recessions. As an illustration of our main empirical result, the top panel of figure 2.1 displays the distribution of firms' employment growth from the Census

[1]With Fatih Guvenen and Nicholas Bloom. For helpful comments and suggestions, we thank John Shea, and seminar participants at Penn Wharton, the 11th World Congress of the Econometric Society (Montreal, 2015), the CESifo Conference on Macroeconomics and Survey Data (Munich, 2015), and the Society for Economic Dynamics Annual Meeting (Toulouse, 2016). Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed. The replication packet for the empirical results of the paper is available on the authors' websites.

Longitudinal Business Dynamics dataset (LBD). The solid line shows the empirical density of employment growth pooling observations from the most recent two recession years, 2001 and 2008. The dashed line instead shows the density for the following expansion years, in this case years 2003 to 2006 and 2010 to 2014. One can clearly see that, relative to expansion periods, the distribution of employment growth during recessions has a thicker left tail, whereas the right exhibits little change, indicating an increase in dispersion that is mostly due to a widening left tail.

The uneven change in the distribution of employment growth from expansion to recession years can be quantified using the Kelley skewness (Kelley, 1947), which is defined as the difference between the 90th-to-50th percentiles spread (a measure of dispersion in the right tail) and the 50th-to-10th percentiles spread (a measure of dispersion in the left tail) divided by the distance between the 90th-to-10th percentiles spread (a measure of the total dispersion of the distribution). For a distribution with a compressed upper half and a dispersed lower half (i.e., a left-tail skew), the Kelley skewness is negative. In the case of the top panel of figure 2.1 we find a decline of the dispersion of employment growth above the median from 0.22 to 0.20 from expansion to recession years whereas the dispersion below the median increases from 0.17 to 0.25. This asymmetric change in the tails generates a decline in the Kelley skewness from 0.10 to -0.12.[2] The bottom panel of figure 2.1 shows a similar pattern for the distribution of sales growth in a sample of publicly traded firms in the United States. As in the case of employment growth, here we also find that recessions are characterized by an increase in dispersion that is mostly accounted for by a widening left tail. Hence, the skewness of the sales growth distribution also drops during recessions.

Our second empirical finding is that skewness of firm-level outcomes is strongly procyclical at the industry-level. That is, within narrow industries groups the skewness of the firm-level employment growth, sales growth, and stock returns are positively correlated with the within-industry economic cycle. Furthermore, using a panel of firms spanning thirty-nine countries we find that the within-country cycle is associated with a decline in the skewness of firm outcomes.

Motivated by the robust empirical evidence that the skewness of firms' growth decreases

[2] Put in a different way, a Kelley skewness of 0.10 indicates that during expansion, 45% of all the dispersion is accounted for by firms with employment growth below the median, whereas during recessions, this share increases to 56%.

during recessions, in the second part of the paper we build a heterogeneous agents model where the key feature is the presence of a large number of entrepreneurs that face shocks with time-varying risk featuring both, time-varying variance and time-varying skewness. In order to capture the potentially non-linear response of firms to shocks, we assume that entrepreneurs are risk-averse, face a combination of convex and non-convex adjustment costs to capital, and can invest in capital and in a risk-free asset. We numerically solve the model and choose the parameters of the firm's productivity process so that our modeled economy matches the average skewness of the sales growth distribution we observe among US firms during expansionary periods and the large decline in the skewness observed during a typical recession. Our results suggest that first-moment shocks combined with risk-aversion and capital adjustment costs, both of which generate asymmetries in the response of firms to shocks, are not sufficient to generate the large swings in the skewness of firm outcomes we document. Hence, in order to match the changes in the skewness of firms outcomes we observe in the data, we consider time-varying skewness in the firms' productivity process. In our main quantitative exercise, we study the aggregate effect of a pure skewness shock: a decline in the skewness of firms' productivity while keeping the mean and variance constant. Our model predicts that a change in the skewness of the distribution of firm-level shocks that matches the decline in the skewness of sales growth we observe among US firms has a negative impact on gross domestic output (GDP) of 1.7%. The decline in aggregate economic activity is quite persistent as GDP stays below its pre-shock level several quarters after the shock. This is in contrast to the standard uncertainty shock analyzed in the literature that typically generates a sharp drop and rapid rebound of GDP. This significant and persistent drop in output is driven by a decline in capital investment, which is the result of three forces. First, the presence of a fixed cost to capital adjustment creates a real options effect that reduces the incentives of firms to invest when skewness declines. Second, the drop in skewness makes capital riskier, inducing an increase in investment in the risk-free asset. Finally, relative to the standard uncertainty shock (a symmetric increase in dispersion), in our model a decline in skewness commands a widening of the left tail of the firm productivity distribution without a corresponding widening of the right tail (an asymmetric increase in dispersion). This change in the skewness cancels out the increase in output generated by an uncertainty shock in models without adjustment costs.

Hence, our results indicate that a negative shock to the skewness of firms' productivity (that keeps the mean and variance constant) can generate by itself a recession.

This paper is related to several strands of literature. First and foremost, our paper relates directly to the study of the effects of uncertainty on firms' decisions. Several papers have shown that an increase in uncertainty can have important macroeconomic implications in the presence of adjustment costs or financial frictions.[3]

Second, several authors have suggested that rare disasters—presumably arising from an asymmetric distribution of shocks—can generate large fluctuations in economic activity, such as the Great Recession. Reviving the ideas introduced first by Rietz (1988), Barro (2006) uses a panel of countries to estimate the probability of large disasters and argue that these low-probability events can have substantial implications for aggregate economic activity and asset pricing.[4] The results of our paper can be seen as evidence that rare disasters also occur at the microeconomic level.

Finally, our paper contributes to a growing literature that focuses directly on the skewness of firms and workers outcomes such as firm productivity (Kehrig (2011)), employment growth (e.g. Ilut et al. (2018) and Decker et al. (2015)), stock returns (e.g. Harvey and Siddique (2000), Kapadia (2006), and Oh and Wachter (2018)), and labor earnings (e.g. Guveven et al. (2014) and Schmidt (2016)).

The rest of the paper is organized as follows. Section 3.2 describes the data we use and the basics statistics discussed in the empirical section. Section 2.3 shows the main empirical results of our paper, that is, that the skewness of several firm-level outcomes is procyclical. Section 2.4 shows the model and section 2.5 shows our quantitative results. Section 2.6 concludes.

[3]See, for example, Arellano et al. (2010), Fernandez-Villaverde et al. (2011), Bachmann and Bayer (2013), Bachmann and Bayer (2014), Gilchrist et al. (2014), Jurado et al. (2015), Leduc and Liu (2016), Basu and Bundick (2017), Berger et al. (2017), Alfaro et al. (2018) and Bloom et al. (2018).

[4]See for instance Gabaix (2008, 2012), Gourio (2008, 2013, 2012), Wachter (2013), Kilic and Wachter (2015), among others.

2.2 Data and Measurement

2.2.1 Data and Sample Selection

Our analysis is based on three large datasets. First, we extract panel data on employment at the firm-level from the Census Bureau's Longitudinal Business Data Base (LBD). The LBD provides high quality measures of employment, wage bill, industry, and firm age for the entire US nonfarm private sector linked over time at the establishment-level from 1976 to 2015. From the LBD we construct employment at the firm and establishment-levels and use it to calculate cross-sectional moments of the distribution of employment growth at narrow firm population groups.

Second, we draw panel data information of publicly traded firms from Compustat, which contains information on sales, employment, stock prices, and other firm-level outcomes. We use data on quarterly sales, daily stock prices, annual sales, and annual employment from 1970 to 2017, and we restrict attention to a sample of firms with more than ten years of data to minimize the types of compositional issues identified in Davis et al. (2006).

Third, we study whether we patterns we document for the United States are also observed in other countries, both developed and developing. To that end, we use cross-country firm-level panel data containing sales and employment information between 1986 and 2016 from the Bureau van Dijk's Osiris dataset. To ensure that changes in the sample of firms do not bias our results, we focus on firms that are present in the sample for ten years or more. Additionally, we restrict our sample to country/year bins with more than one hundred firms, countries with at least ten years of data, and years with five countries or more. Our main results are based on an unbalanced panel of firms spanning forty countries from 1991 to 2015. We complement this dataset with information on firm-level stock prices obtained from the Global Compustat dataset. Applying similar selection criteria, we obtain a sample of daily stock price information for firms in 29 countries from 1985 to 2017. Table 2.1 summarizes the data sources and the main sample characteristics. Additional details on data construction, selection criteria, and moment calculation can be found in Appendix B.1.[5]

[5] The online appendix–available at the authors' websites–provides additional details about the underlying data and describes the material to replicate the results presented in the empirical section of the paper.

2.2.2 Measuring Skewness

For most of our results, we measure the growth rate of a firm-level outcome as the log-difference between period t and $t+k$ where t is a quarter for stock returns, and a year in the case of employment and sales. Our basic measure of dispersion is the cross-sectional spread between the 90th and the 10th percentiles, denoted by $P9010_t$, where t is a quarter or a year depending on the dataset. The $P9010_t$ is used (rather than, for example, the standard deviation of growth rates) for robustness to outliers which, are common in firm level micro datasets. Additionally, we use the difference between the 90th and 50th percentiles, denoted by $P9050_t$, and the difference between the 50th and 10th percentiles, denoted by $P5010_t$, as measures of dispersion in the right and left tails of the distribution respectively. Finally, our preferred measure of skewness is the Kelley skewness, which is defined as

$$KSK_t = \underbrace{\frac{P90_t - P50_t}{P90_t - P10_t}}_{\text{Right Tail Share}} - \underbrace{\frac{P50_t - P10_t}{P90_t - P10_t}}_{\text{Left Tail Share}} \qquad \in [-1, 1]. \tag{2.1}$$

Relative to the third standardized moment (which is another measure of skewness), the Kelley skewness has the advantage of being robust to outliers and provides a simple decomposition of the share of total dispersion that is accounted for by the left and the right tails of a distribution.[6] A negative value of the Kelley skewness indicates that the left tail accounts for more than one-half of the total dispersion and the distribution is negatively skewed. In the same way, a positive value indicates a positive skewed distribution, with a right tail that accounts for the largest share of the total dispersion. Clearly, this measure is equal to zero if the distribution is symmetric, such as for the Normal distribution.

2.3 Skewness over the Business Cycle

In this section we show that the distribution of firm-level growth has a larger left tail in recessions in both the United States (section 2.3.1), across countries (section 2.3.2), and

[6] An important drawback of this measure of skewness is that it is invariant to 20% of the observations in the sample (the top and bottom 10% of the distribution). Alternatively, one could write KSK_t by using the 95th and 5th percentiles of the distribution. Our main results, however, are not sensitive to changes in the percentiles used to calculate KSK_t. Additional measures of skewness can be found in Kim and White (2004).

then confirm our results hold within industries (section 2.3.3).

2.3.1 Firm Skewness over the Cycle

The first contribution of our paper is to show that the skewness of the growth rates of firm–level outcomes varies over time and is strongly procyclical, declining substantially during recessions and rising in booms. We start by considering the evolution of the Kelley skewness of the distribution of the growth rate of firm employment for a sample of firms from the Census's LBD which is displayed in the top panel of figure 2.2. To calculate Kelley skewness we weight observations by firm's employment so that our measure reflects the underlying firm-size distribution.[7] Figure 2.2 shows, first, that the skewness of employment growth, in average, is positive and around 10% for most the sample period. Second, the skewness of employment growth is strongly procyclical, declining from an average of 11% at the peak of the typical recession to around -10% at the trough, that is, a drop of 21 percentage points. Similarly, the bottom of figure 2.2 shows the cross sectional skewness of annual sales growth for a sample of publicly traded firms from Compustat. Relative to our sample from LBD, this is a more selective set of mostly large firms. Still, we find that the skewness of the distribution of sales growth is positive on average, and declines around 20 percentage points during a recession.

The decline in the skewness of firm growth occurring during recessions is driven by a rapid change in the relative weight of the tails of the distribution. This can be observed in the top panel of figure 2.3 where we plot the spread between the 50th and the 10th percentiles of the employment growth ($P5010_t$, black solid line) and the 90th and the 50th percentiles spread ($P9050_t$, blue dashed line). The bottom panel of figure 2.3 shows the same set of statistics for sales growth. Two important aspects are worth noticing. First, during expansionary periods, the right tail outweighs the left tail ($P9050_t$ is most of the times above the $P5010_t$), generating a distribution of firm's outcomes that is positively skewed. Second, both for employment and sales growth, recessions are episodes in which the $P5010_t$ expands, indicating a left tail that stretches out, whereas the right tail shrinks. This uneven

[7] In particular, we weight the employment growth of firm i in period t by the average employment in periods t and $t+1$, that is $\overline{E}_{i,t} = 0.5 \times (E_{i,t} + E_{i,t+1})$. The results for publicly traded firms are un weighted since most of the firms are large.

change of the tails drives a drop in the skewness of firms' employment and sales growth.[8] To have a better sense of the magnitude of the change in the skewness and its relation with the cycle, the left panel of table 2.2 shows a set of time-series regressions where the dependent variable is the Kelley skewness of the cross-sectional distribution of different firm-level outcomes for the United States. In all regressions, the independent variable is the growth rate of real GDP per capita, which we have normalized to have unitary variance so the coefficients are comparable across columns. Column 1 shows that a change of one standard deviation in GDP per capita is associated with a change in the skewness of firms' employment growth of 4.6 percentage points. Columns 2 and 3 show similar results for the skewness of sales growth and stock returns.[9]

2.3.2 Firm Skewness Across Countries

Are the cyclical properties of the skewness of firm-level outcomes a characteristic of the US economy or are observed across a broader set of countries? To answer this question we use firm-level panel data across different countries to show that the skewness at the microeconomic level positively co-moves with the country business cycle.

The top panel of figure 2.4 displays the empirical density of the distribution of the growth rate of annual real sales (in US dollars as of 2005) for a panel of firms spanning across thirty nine countries from 1991 to 2015. The solid red line is the density of the growth rate of sales during recession periods, where a recession is defined as a year in which the growth rate of GDP is in the first decile of the country-specific GDP growth distribution. The dashed black line is the density of sales growth during expansion periods defined as years in which GDP growth is above the first decile of the country-specific distribution of GDP growth. Similar to the results presented in figure 2.1, the dispersion of the sales growth distribution increases little during recession years as the difference between the 90th and the 10th percentiles of the distribution widens from 0.82 to 0.85. The left tail

[8]Figure B.1 shows a similar set of results for the annual change of real sales at the quarterly level.

[9]Table B.3 in appendix B.2 shows that the skewness of firm-level outcomes remains strongly procyclical if we residualize the firms' outcomes by firm's observable characteristics and fixed heterogeneity, if we consider the growth rate of sales-per-worker– more closely related to firm productivity– and if we look at the three year growth rate of firm's outcomes (appendix table B.1). We also confirm that the dispersion of firms outcomes is countercyclical (appendix table B.2) but we do not find significant business cycle variation in the Kurtosis of firm's outcomes (right panel of table B.3).

of the distribution, however, stretches out with a corresponding increase in the spread between the 50th and 10th percentiles from 0.36 to 0.43. At the same time, right tail shrinks, as the spread between the 90th and 50th percentiles which declines from 0.46 to 0.43. Consequently, the Kelley skewness drops from 0.12 to 0.00.

To have a clearer picture of the relation between the within-country skewness of firm outcomes and the business conditions, the bottom left panel of figure 2.4 shows a scatter plot in which the x-axis is the average of firm-employment growth within a country-year bin whereas the y-axis is the Kelley skewness of the same firm-level outcome. This figure indicates that when the within-country average firm employment growth is -15% –typically during a recession– the Kelley skewness is -30%, which means that most of the mass of the distribution of employment growth (actually, a 65% of the total dispersion) is accounted for by the left tail. In contrast, when the average employment growth is 10%, the skewness is 30%, indicating that 65% of the total dispersion is accounted for by the right tail of the distribution. The bottom right panel of figure 2.4 shows similar results for the within-country sales growth distribution. Importantly, to construct these figures we have controlled for country- and time-fixed effects, therefore, the results are not driven by fixed characteristics of the countries considered in the sample or aggregate shocks–such as the Great Recession–that can affect all countries at the same time.[10]

Finally, the center panel of table 2.2 exploits our cross-country data to evaluate more systematically the relation between firm-level skewness and aggregate economic conditions. Column (4) shows a country panel regression in which the dependent variable is the cross-sectional skewness of the employment growth across all firms within the country. The business cycle is captured by the growth rate of GDP per capita which is again normalized to have unit variance so the results can be directly compared with those obtained using data from the United States. The regression also includes a full set of time- and country-fixed effects to control for aggregate economic conditions that might affect all countries or fixed differences across countries. Here we also find strong procyclical skewness for employment growth, sales growth, and firm's stock results. This further confirms that the decline in

[10]One important concern is that our cross country results are based on exclusively on publicly traded firms. Interestedly, we also find remarkably similar results if we consider a unbalanced panel of firms (private and publicly traded) drawn from the BvD Amadeus dataset as figure B.2 shows. The BvD Amadeus dataset covers a shorter period of time (2000 to 2015 for most countries) over a smaller sample of European countries.

the skewness of firm-level outcomes is a robust feature of the business cycles.

2.3.3 Firm Skewness Across Industries

Our second empirical finding is that the skewness of firm-level outcomes is strongly procyclical within industries. We show this result by using our sample of firms from LBD. The top panel of figure 2.5 shows a bin scattered plot between the average employment growth within an industry-year bin and the cross-sectional Kelly skewness of employment growth within the same group. In this case, a positive correlation between the average and the skewness of employment growth indicates that periods of low economic activity at the *industry-level* are associated with a distribution of employment growth that is negatively skewed, whereas periods of high industry economic activity are associated with a positively skewed distribution. In terms of magnitudes, the top panel of figure 2.5 shows that when the industry employment growth is -8%, the Kelley skewness is around 20%, indicating that 60% of the total dispersion of employment growth is accounted for by the left tail of the distribution. When the average employment growth is 8% instead, the Kelley is skewness is 20%, indicating that the right tail accounts for 60% of the total dispersion. Similarly, the bottom panel of figure 2.5 shows that the within industry skewness of sales growth is higher when the within industry average sales growth is higher. Hence, sectors that grow faster are also sectors in which the skewness of firm-level outcomes is higher.[11]
We then use firm-level data from a sample of firms from Compustat to examine the relation of the industry cycle and the skewness of sales growth, employment growth, and quarterly returns within NAIC 2-digit industry-period bins. Columns 7 to 9 of table 2.2 display a series of industry panel regression in which the dependent variable is the Kelley skewness of the growth rate of different firm-level outcomes across all firms within an industry-period bin. The specification we run is,

$$KSK_{j,t} = \alpha_j + \gamma_t + \beta \Delta S_{j,t} + \epsilon_{j,t}, \tag{2.2}$$

where $KSK_{j,t}$ is the Kelley skewness of the annual growth for firms in sector j in period t, α_j is an industry fixed effect, and γ_t is a period fixed effect. The independent variable in

[11] Appendix figure B.3 shows remarkably similar results for other firm's outcomes such as three-years sales growth, three-year employment growth, and stock returns.

each regression, $\Delta S_{j,t}$, is a measure of the industry cycle which we proxy with the within-industry average growth rate of sales. Here we have re scaled the real sales growth within each sector to have a variance of one so that the regression coefficient can be interpreted as the effect of a change in the within industry sales growth of one standard deviation and can be directly compared to the coefficients of columns 1 to 2 of table 2.2. Importantly, we also include a full set of time and industry fixed-effects, so that the results will be driven by industry rather than aggregate changes in growth rates.[12]

Column 7 of table 2.2 shows that the skewness of employment growth is significantly lower during industry slowdowns. Specifically, a one standard deviation decline in the within industry average sales growth is associated with a decline in the skewness of employment growth of 7 percentage points. This is almost two times larger than the effect of a change in one standard deviation in GDP growth on the skewness of employment growth across all firms in the economy. Similarly, a one standard deviation decline in average sales growth is correlated with a decline of 13 percentage points in the skewness of sales growth, and a decline of 1.6 percentage points in the skewness stock returns. Again, it is worth noting that these regressions include a full set of period and industry dummies, the relation between skewness and the business cycle is independent of the aggregate economic conditions.

2.3.4 Robustness

In this section, we perform several robustness checks using the large sample size of the LBD. First, we study whether the skewness of employment growth declines within firms' groups defined by age and size, or if we look at establishments instead of firms. We then expand the notion of skewness to account for a larger proportion of the distribution of employment growth, and finally, we study whether including the entry and exit of firms change the observed evolution of the skewness during recessions.

First, as we show in the top panels of figure 2.6, the skewness of employment growth is procyclical for firms within different employment size groups. The middle left panel shows that the skewness of employment growth of young firms (those of five years old or less)

[12] We find a similar positive and statistically significant relationship between industry cycles and skewness when we consider each industry separately. Appendix figure B.4 shows the coefficient of a set of within-industry time-series regressions of the Kelley skewness of firms' growth on the within-industry average firm growth. Notice that there is substantial heterogeneity across industries and for all of them the coefficient on the average firm growth is economically and statistically significant.

is larger than the skewness for the rest of the firms the economy which relates to the fact that young firms must grow fast in order to survive (Haltiwanger et al. (2016)). The skewness, however, drops during recessions across all age categories. The middle right panel of figure 2.6 shows that the skewness of employment growth is also procyclical at the establishment-level, indicating that our results are not driven by a small number of large firms.[13]

Second, we vary our preferred measure of skewness. By considering the 90th and the 10th percentiles we are effectively dropping twenty percent of the distribution, which is potentially important, as our results hinge on the differential response of the tails of the distribution of firm-level outcomes to aggregate economic conditions. We can modify the Kelley skewness in equation 2.1 to reduce the proportion of the sample left out at the tails. In particular, we calculate the skewness considering the 95th and 5th percentiles of the distribution, and as a third measure, we consider the 2.5th and 97.5th percentiles. The bottom left panel of figure 2.6 shows that decreasing the proportion of the distribution left out of the sample does not change substantially the cyclical properties of the skewness of the distribution of employment growth.

Third, our main results are based on the distribution of employment growth calculated as the log-change of firm employment. If a firm exits the market due to a change in aggregate economic conditions or a new firm enters, our measure of growth rate, and consequently, the skewness of the distribution, will not take them into account. To solve this problem we calculate the skewness of the employment growth distribution considering the arc-percent change of employment. The arc-percentage change is defined as $2\left(x_{i,t+k}-x_{i,t}\right)/\left(x_{i,t+k}+x_{i,t}\right)$. This measure has been popularized in the firm dynamics literature by Davis and Haltiwanger (1992) and has the advantage that, while it is similar to a percentage change, it allows for entry/exit by including both time t and $t+k$ measures in the denominator, one of which is allowed to be zero.[14] The bottom right panel of figure 2.6 shows that the cyclical properties of the skewness of employment growth do not change

[13]Furthermore, appendix figure B.5 shows that the skewness of employment growth is also procyclical within establishment groups defined by establishment size and age.

[14]Notice that, for a firm with a positive value of $x_{i,t}$ which is inactive in period $t+k$, and henceforth has a value of $x_{i,t+k}$ equal to 0, the arc-percent change takes the value of -2. Similarly, for an entering firm (that is, $x_{i,t}$ is equal to 0 but $x_{i,t+k}$ is positive) the arc-percent change takes the value of 2.

substantially when accounting for the entry and exit of firms.[15]

In summary, we have shown that the skewness of firm-level outcomes declines sharply during recessions, both at the aggregate and at the industry level. Motivated by this robust evidence, in the next section we study a heterogeneous agents model that we use to evaluate the macroeconomic importance of the large swings in the skewness we observe in the data.

2.4 Model

Given the robust evidence presented in the previous section, a natural question is to ask whether these uneven changes in the distribution of firm-level outcomes have aggregate economic implications. To answer this question in this section we analyze the quantitative impact of a variation in the skewness on firm-level shocks in the context of a heterogeneous agents model. Specifically, we consider an economy populated by a large number of households/entrepreneurs that have access to a production function that uses capital and labor to produce a homogeneous good. Entrepreneurs can save in capital and in a risk-free asset that pays a fixed return. Crucially, the shape of the distribution of idiosyncratic shocks changes with aggregate business conditions so as to match the time-varying nature of the distribution of firm-level outcomes. Furthermore, in order to capture potential non-linearities in the response of entrepreneurs to aggregate and idiosyncratic shocks, we assume that entrepreneurs are subject to capital adjustment costs.

2.4.1 Entrepreneurs

Production Technology

The economy is populated by a large number of infinitely lived heterogeneous households that use capital and labor inputs to produce a homogeneous good by means of a decreasing returns to scale production function. Specifically, each entrepreneur j produces output according to,

$$y_{j,t} = A_t e_{j,t} k_{j,t}^{\alpha} n_{j,t}^{\nu}, \text{ with } \alpha + \nu < 1.$$

[15] Our results are also robust when we look at firms of different size and age, and for the growth of establishments rather than firms. All these results are based on a sample of firms from Census' LBD and are under disclosure process.

The aggregate productivity shock, denoted by A_t, follows a standard first-order autoregressive process, given by

$$\log A_t = \rho_A \log A_{t-1} + \sigma_\eta \eta_t,$$

where η_t is a Gaussian innovation with zero mean and unitary variance. We assume that the idiosyncratic process $e_{j,t}$ is given by,

$$e_{j,t} = \rho e_{j,t-1} + \epsilon_{j,t}, \tag{2.3}$$

where the innovation $\epsilon_{j,t}$ is assumed to have zero mean, time-varying variance, denoted by $\sigma_{\epsilon,t-1}$, and time-varying skewness, denoted by $\gamma_{\epsilon,t-1}$. Notice we have assumed that the distribution of innovations in period t depends on the values of the variance and skewness observed in period $t-1$. Hence, an increase in risk, in the form of an increase in dispersion or a decrease in the skewness of firms' shocks, represents news about the characteristics of the distribution of innovations in the future but not a change in the distribution from which the current realizations of $\epsilon_{j,t}$ are drawn.

Capital Adjustment Costs

It is possible that the distribution of firm-level outcomes changes asymmetrically over the business cycle because of adjustment costs or other rigidities that distort firms' responses to shocks. In other words, the distribution of firm-level outcomes, such as sales growth or employment growth, can change asymmetrically because of the endogenous response of firm to symmetric shocks.[16] In order to capture this channel, we consider a combination of convex and non-convex adjustment cost to capital. In particular, we assume that physical capital depreciates at the rate δ and adjustment costs are equal to the sum of a fixed disruption cost, ϕ_1, which the entrepreneur pays for any investment or disinvestment, a quadratic adjustment cost, ϕ_2, and a resale cost for disinvestment, ϕ_3. The adjustment cost function for capital input is then given by,

$$\phi\left(k_{j,t+1}, k_{j,t}\right) = \phi_1 \mathbb{I}_{|i_{j,t}|>0} y_{j,t} + \frac{\phi_2}{2}\left(\frac{i_{j,t}}{k_{j,t-1}}\right)^2 + \left(1-\phi_3\right)\left|i_{j,t}\right| \mathbb{I}_{i_{j,t}<0}, \tag{2.4}$$

[16] Ilut et al. (2018), for instance, argues that a left skewed distribution of employment growth is the results of an asymmetric response of firms to positive and negative shocks.

where $i_{j,t}$ is the entrepreneur's investment in capital given by

$$i_{j,t} = k_{j,t+1} - (1-\delta)\, k_{j,t}. \tag{2.5}$$

The Problem of the Entrepreneur

Denote the entrepreneur's consumption by $c_{j,t}$ which is valued by the utility function $u(c_{j,t}) = c_{j,t}^{1-\lambda}$. Entrepreneurs do not value leisure and supply one unit of labor, which they use for running their own firm (they cannot work for someone else's firm). They can save in capital and in a risk-free asset that pays an interest rate r_t. Denote the entrepreneur's value function by $V(k_{j,t}, a_{j,t}, e_{j,t}; \omega_t)$ where $k_{j,t}$ is the entrepreneur's stock of physical capital, $a_{j,t}$ is the beginning-of-the-period holdings in the risk-free asset, and $e_{j,t}$ is the level of her idiosyncratic productivity. For notational simplicity, define the vector of aggregates states as $\omega_t \equiv (A_t, \sigma_{\epsilon,t-1}, \gamma_{\epsilon,t-1}, \mu_t)$ where A_t is the level aggregate productivity, $\sigma_{\epsilon,t-1}$ and $\gamma_{\epsilon,t-1}$ are the variance and the skewness of the distribution of idiosyncratic shock respectively, and μ_t is the distribution of entrepreneurs over idiosyncratic states. Then, we can write the problem of the entrepreneur as,

$$V(k_{j,t}, a_{j,t}, e_{j,t}; \omega_t) = \max_{\substack{c_{j,t},\, k_{j,t+1},\\ a_{j,t+1},\, n_{j,t}}} \left(c_{j,t}^{1-\lambda} + \beta \mathbb{E}\left[V(k_{j,t+1}, a_{j,t+1}, e_{j,t+1}; \omega_{t+1})^{1-\xi} \right]^{\frac{1-\lambda}{1-\xi}} \right)^{\frac{1}{1-\lambda}}, \tag{2.6}$$

$$\begin{gathered}
\text{s.t. } c_{j,t} + i_{j,t} + a_{j,t+1} \leq y_{j,t} - w_t n_{j,t} - \phi(k_{j,t+1}, k_{j,t}) + (1+r_t)\, a_{i,t}, \\
i_{j,t} = k_{j,t+1} - (1-\delta)\, k_{j,t}, \\
\mu_{t+1}(k_{j,t+1}, a_{j,t+1}, e_{j,t+1}) = \Gamma(\omega_t), \\
k_{j,t} > 0, a_{j,t} \geq 0, n_{j,t} > 0,
\end{gathered}$$

given the laws of motion for A_t, $\sigma_{\epsilon,t}$, and $\gamma_{\epsilon,t}$. In this specification, ξ is the coefficient of risk aversion and λ is inversely related to the inter-temporal elasticity of substitution. The term $w_t \equiv w(\omega_t)$ denotes the wage rate in the economy. In what follows, we assume the interest rate on the risk-free asset is fixed, that is $r_t = r(\omega_t) = r$.[17] Let $C^e(k_{j,t}, a_{j,t}, e_{j,t}; \omega_t)$, $K^e(k_{j,t}, a_{j,t}, e_{j,t}; \omega_t)$, $N^e(k_{j,t}, a_{j,t}, e_{j,t}; \omega_t)$, and $A^e(k_{j,t}, a_{j,t}, e_{j,t}; \omega_t)$, denote the policy rules of consumption, next's period capital, current period labor, and risk-free asset for the entrepreneurs.

2.4.2 Non-Entrepreneurial Households

The economy is populated by a large number of identical hand-to-mouth household that consume C_t units of the homogeneous good and supply labor elastically which we denote by N_t. In concrete, we assume that the non-entrepreneurial households solve the static problem,

$$U(C_t, N_t) = \max_{C_t, N_t} \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1-\gamma}}{1-\gamma} \right\}, \tag{2.7}$$

$$C_t \leq w_t N_t,$$

given the law of motion of the aggregate state, ω_t. Denote by $C(\omega_t)$ and $N(\omega_t)$ the optimal choices of consumption and labor for the non entrepreneurial household.

2.4.3 Recursive Competitive Equilibrium

Given the exogenous process for the aggregate productivity, A, the exogenous process of the variance and skewness of e_j, an interest rate of the risk-free asset, r, and the evolution of the idiosyncratic productivity processes for the entrepreneurs, $\{e_j\}_{j \in J}$, a recursive competitive equilibrium for this economy is a set of policy functions $\left\{ \left\{ C_j^e, K_j^e, N_j^e, A_{j,}^e \right\}_{j \in J}, C, N \right\}_{t=0}^{\infty}$, a wage function $\{w\}$, and value functions $\{V, U\}$ such that i) the policy and value functions solve (2.6) and (2.7) respectively, ii) labor market clears, that is

$$\int N^e(k_j, a_j, e_j; \omega)\, d\mu(k_j, a_j, e_j) = N(\omega),$$

[17] This imply that we will not solve the interest rate in equilibrium. The wage rate, however, is such that the labor market clears.

and iv) the mapping $\Gamma(\omega)$ that determines the evolution of the joint distribution of e_j, k_j, and a_j is consistent with the policy functions, the evolution of the aggregate productivity process, and the evolution of the process of σ_ϵ and γ_ϵ.

2.4.4 Parameters, Estimation, and Model Fit

In this section, we describe the quantitative specification of our modeled economy. To solve the entrepreneurs' problem we employ non-linear methods similar to Krusell and Smith (1998). Most of our parameters are standard in the macro literature and we take them from the existing estimates when possible. However, the parameters governing the stochastic process of productivity are novel to our analysis and we use a simulated method of moments approach to estimate them.

Frequency and Preferences

We set the time period to a quarter. For the entrepreneurs, we set ξ, the risk aversion coefficient, equal to 6.0 and $1/\lambda$, the elasticity of substitution, to $1/\lambda = 0.2$, which are in the midpoint of the values used in Guvenen (2009). The household's discount rate, β, is set to $0.95^{0.25}$, whereas the interest rate on the risk-free asset is set to match an annual return of 2%. For the non-entrepreneurial sector, we set σ to 2. For the labor supply of the non-entrepreneurial households, we fix a value of γ to 1.5 and we choose ψ so that they spend an average of 33% of their time working.

Production Technology and Adjustment Costs

The exponents of the capital and labor inputs in the entrepreneur's technology are set to $\alpha = 0.25$ and $\nu = 0.5$. The capital depreciation rate, δ, is set to match a 14% of annual depreciation. As for the adjustment cost parameters, we set the fixed adjustment cost of capital, ϕ_1, equal to 1.5%, a quadratic adjustment cost, ϕ_2, equal to 7.0, and a resale cost, ϕ_3, equal to 34.0%.

Aggregate Productivity

We assume that the aggregate productivity follows a standard first-order autoregressive process with autocorrelation of 0.95 and normally distributed innovations with mean 0 and standard deviation of 0.75%, similar to the quarterly values used in other papers in the literature. Table 2.3 summarizes the set of calibrated parameters.

Idiosyncratic Productivity

To capture time-varying risk we assume that the economy transitions between in two states. The first, which we denote as low risk state, corresponds to periods where the variance of the innovations of the idiosyncratic shocks is low, $\sigma_{\epsilon,t} = \sigma_L$, and the skewness is positive, $\gamma_{\epsilon,t} = \gamma_H$, as we observe in non-recession periods. The second state, or high risk state, corresponds to periods of high dispersion, $\sigma_{\epsilon,t} = \sigma_H$, and negative skewness, $\gamma_{\epsilon,t} = \gamma_L$, as we observe during a typical recession. Low and high risk states alternate following a first-order Markov process. To capture the potential non-gaussian nature of the idiosyncratic shocks we assume that, conditional on the values of $\sigma_{\epsilon,t}$ and $\gamma_{\epsilon,t}$, the innovations in 2.3 are drawn from a mixture of two normally distributed random variables, that is,

$$\epsilon_{j,t} \sim \begin{cases} N\left(\mu^s, \sigma_1^s\right) & \text{with prob } p^s, \\ N\left(-\frac{p^s}{1-p^s}\mu^s, \sigma_2^s\right) & \text{with prob } 1-p^s, \end{cases} \tag{2.8}$$

where s can be a high or low risk state. Hence, in order to fully characterize the stochastic process faced by firms we need to find ten parameters, namely, $\{\mu^s, \sigma_1^s, \sigma_2^s, p^s\}$ with $s \in \{H, S\}$, and the parameters governing the transition probabilities between low and high risk periods, denoted by π_L and π_H respectively.

Since we do not directly observe the productivity process faced by the firms, we choose the parameters of the stochastic process of firm's productivity of our model to match the main features of the US data described in the empirical section of the paper. In particular, we take data of quarterly sales growth from a sample of publicly traded firms, and we search for parameters of the stochastic process so that the cross sectional distribution of sales growth derived from the model reproduces the observed average values of the 90th-to-50th percentiles spread, the 50th-to-10th percentiles spread, the Kelley Skewness, and the

90th-to-10th percentiles spread during expansion periods and the same set of moments for recession periods for a total of eight moments of the quarterly sales growth distribution.[18] The probability of being in the high risk state in the next period conditional on being in the high risk state in this period, π_H, is set to be equal to the fraction recession quarters that are followed from another recession quarter in the data, $\pi_H = 0.84$, whereas the transition probability of the low risk state, π_L, is set so that the share of expansion quarters following another expansion quarter is 0.95. Recession and expansion periods in the data correspond to the recession quarters defined by the NBER from 1970 to 2014.

Based on our estimations, we find that in periods of low risk, the variance of the idiosyncratic productivity shocks, η, is equal to 4.85% whereas the skewness is equal to 0.85. In contrast, in periods of high risk, the variance of the productivity shocks is equal to 6.85% and the skewness is negative and equal to -1.14. Table 2.6 displays our estimates for the different parameters of the idiosyncratic productivity process whereas table 2.4 shows the targeted and model-simulated moments.[19]

2.5 Quantitative Results

In this section, we study the quantitive implications of our model. We first analyze standard business cycle statistics. Then, in our main quantitative exercise, we evaluate the response of our modeled economy to a shock that increases risk by reducing the skewness of idiosyncratic productivity while keeping the mean and variance constant. Finally, we compare the response of our model after a variance shock (i.e., a standard uncertainty shock which implies a symmetric increase in dispersion) to a negative skewness shocks (i.e., an asymmetric increase in dispersion), and then to a combined shock of dispersion and skewness (i.e., a change in risk that resembles what happens in a typical recession).

[18] Appendix figure B.1 displays the evolution of the cross-sectional dispersion and skewness of the sales growth distribution for our sample of publicly traded firms from Compustat at the quarterly frequency.

[19] The variance of a random variable η which is distributed as a mixture of two normally distributed random variables is given by $Var(\eta) = \mathbb{E}\left(\eta^2\right) - \mathbb{E}(\eta)^2$ whereas the skewness is given by $Skew(\eta) = \left(\mathbb{E}\left(\eta^3\right) - 3\mathbb{E}(\eta)\, Var(\eta) - \mathbb{E}(\eta)^3\right)/Var(\eta)^{\frac{3}{2}}$. Here $\mathbb{E}(\eta)$ is the first moment of the η given by $\mathbb{E}(\eta) = p_1\mu_1 + p_2\mu_2$. Similarly, $\mathbb{E}(\eta)^2 = p_1\left(\mu_1^2 + \sigma_1^2\right) + p_2\left(\mu_2^2 + \sigma_2^2\right)$ and $\mathbb{E}\left(\eta^3\right) = p_1\left(\mu_1^3 + 3\mu_1\sigma_1^2\right) + p_2\left(\mu_2^3 + 3\mu_2\sigma_2^2\right)$ are the second and third moments.

2.5.1 Business Cycle Statistics

Table 2.7 shows a set of standard business cycle statistics generated from our modeled economy. To obtain these statistics we simulate our economy for 5,000 periods and we discard the first 500. We then calculate the standard deviation and correlation with aggregate output for several aggregate time series. All statistics are in the neighborhood of what is observed in the data: investment is more volatile than output whereas consumption is less volatile. Additionally, our model generates an average annual risk premium of 5.3%, which is in line with the empirical estimates based on US data. We conclude that our model is consistent with the standard business cycle statistics found in the literature.

2.5.2 Idiosyncratic Shocks and Model Fit

To evaluate the effects of a decrease in the skewness of firm-level shocks, we independently simulate 1,000 economies, each of 300 quarters length. For the first 150 periods, the economy remains in the low-risk state, then all economies are hit by a change in the level of risk (i.e. a decrease in the skewness of firm-level shocks, an increase in dispersion of firm-level shocks, or both at the same time). From that period on, all economies evolve normally. We then average different macroeconomic outcomes across all simulated economies and we calculate the impact of the change in risk as the percentage deviation of a given macro variable relative to its value in the period previous the shock.

Comparing the impact of a change in risk that combines dispersion and skewness to a case in which either skewness or dispersion change while keeping the mean of the productivity shocks constant is key for the quantitative analysis we perform in the next section. Hence, before analyzing the effect on the macroeconomic aggregates it is informative to study the evolution of the distribution of idiosyncratic shocks experienced by the firms after a risk shock. In particular, we must make sure that our model can separate a change in dispersion from a change in the skewness of shocks without impacting the average productivity of the firms, so that our results are not driven by changes in the first moment of the productivity distribution, but only by changes in either the dispersion and or the skewness.

The left row of figure 2.7 displays moments of the distribution of firm's idiosyncratic productivity growth, $\Delta e_{j,t} = e_{j,t} - e_{j,t-4}$, for three cases. In the first, the economy moves from the low-risk state to the high-risk state leading to an increase in the variance and a decrease in the skewness of idiosyncratic shocks (blue line with circles) which corresponds to what is observed during a typical recession. In the second case, the increase in risk leads only to a decrease in the skewness of idiosyncratic shocks (black line with diamonds), and finally, in the third case, the increase in risk leads to an increase in the variance of idiosyncratic shocks only which is the typical uncertainty shock studied in the literature (red line with triangles).[20] The top left panel of figure 2.7 shows that the average firm in our model does not experience a change in firm-level productivity when risk changes. This ensures that our results are not driven by a change in average firm productivity. Then, comparing the black line in the middle and bottom left panels one can see that our model is able to generate a pure change in the skewness, that is, a change in the productivity distribution that reflects only a decrease in the skewness but a muted change in the mean and the variance of the firm-level productivity distribution.[21] Similarly, our model can generate a pure uncertainty shock (the red line with triangles in the middle panel of figure 2.7).

It is also important to analyze the impact of the change in risk on the sales growth distribution. The right row of figure 2.7 shows the average, the dispersion, and the skewness of the annual change in quarterly sales implied by the model calculated as $\Delta y_{j,t} = \log y_{j,t} - \log y_{j,t-4}$. It is not surprising that a change in risk that combines a simultaneous increase in the variance and a decrease in the skewness of firm-level productivity shocks generates an increase in the cross-sectional dispersion of sales growth and a large decrease in skewness (blue line with circles in the middle and bottom right panels). Comparing the case in which only dispersion changes–which is the typical uncertainty shock–to the case in which only the skewness changes–the baseline case we discuss in the following section–one can see that by considering a shock with time-varying skewness the model is able to capture the asymmetric response of the tails of the sales growth distribution (compare the red line with triangles to the blue line with circles in the bottom right

[20] To make this comparison, we reestimate the parameters of the stochastic process in 2.8 to separate the changes in dispersion (a symmetric increase in risk) from changes in dispersion and skewness (an asymmetric increase in risk). Table 2.5 shows the estimation targets for each case.

[21] The median firm, however, experiences an increase in productivity after a decline in the skewness that keeps the mean and variance constant. This increase in productivity goes against our results as our model predicts a negative aggregate response of the economy to a drop in skewness.

panel). Moreover, the model generates a drop in Kelley skewness of around 20 percentage points which is in line with the drop observed during recession periods in the United States. This is the first results of our quantitive analysis: in the context of a model with adjustment cost to capital and risk-averse entrepreneurs, a pure uncertainty shock does not generate the large asymmetric changes in the sales growth distribution that we document in section 2.3.[22] Notice also that the average sales growth greatly responds to a change in the risk conditions in the economy (left bottom panel) but this response is only driven by the endogenous capital and hiring response of firms to a change in the risk conditions as the average productivity growth is unaltered.

2.5.3 The Macroeconomic Effect of a Skewness Shock

In this section, we analyze the macroeconomic effect of a decrease in the skewness of firm-level productivity. For doing that, we shock the economy with a change in the skewness of the innovations of $e_{j,t}$ and we calculate the response of different macroeconomic aggregates as the percentage change relative to their value prior the shock. In our exercise, when the economy receives a skewness shock that drives the skewness from γ_H to γ_L, we keep the mean and variance of the idiosyncratic productivity constant at their low-risk level so our results reflect a pure change in the skewness of the distribution. Moreover, our timing assumption implies that in the period the shock arrives, the change in the skewness only represents news about the future economic conditions as the realizations of the productivity process that firms experience are drawn from a distribution with skewness equal to its pre-recession values.

Figure 2.8 shows that output declines by 1.4% four quarters after a skewness shock and 1.7% after eight quarters. This is a significant decline in aggregate economic activity considering that only the shape of the distribution of firm-level shocks has changed. Moreover, the decline in output is quite persistent, staying below its pre-shock level even after twelve periods after the shock. This is in contrast with the typical uncertainty shock that generates a decrease in output and a rapid rebound few quarters after the shock. In our model, the

[22]Figure B.5c in the appendix shows that the dispersion and skewness of sales growth do not respond to a shock to aggregate productivity, A_t, neither. Furthermore, as it is shown in figure B.5a, a change in the skewness of firm's shocks generates a persistent decline in the skewness of employment growth and a decline in the skewness of three-years sales growth.

drop in output is generated by the rapid and persistent decline in capital investment after a change in skewness. The top right panel of figure 2.9 shows that capital investment drops around 15% during the first quarter after the shock and stays below its pre-shock level for several quarters. Labor does not drop in the first period after the shock because labor is fully flexible and news about the future conditions of risk do not change firms' hiring decisions.[23] In contrast, consumption declines rapidly in response to the decrease in the skewness of firm-level shocks, dropping around 1% relative to its pre-shock level, whereas the accumulation of risk-free asset increases because capital is now riskier.

Importantly, in the first quarter after the shock, the response of investment and consumption is not driven by a change in the skewness of the realizations of $e_{j,t}$ received by the firms–recall our timing assumption in equation 2.3–but by a change in the perception about the risk in the economy: at the moment of the shock, entrepreneurs receive news that in the future the distribution of $e_{j,t}$ will be left skewed and their endogenous responses drive a decline in investment and consumption. A decrease in skewness triggers a precautionary increase on entrepreneur's savings, but since capital is riskier, investment in the risk-free asset surges as it is shown in the bottom right panel of figure 2.9. We conclude that a decline in the skewness of the distribution of idiosyncratic shocks can by itself generate a persistent drop in aggregate economic activity.

2.5.4 Variance and Skewness Shocks

Our empirical evidence indicates that a typical recession is characterized by an asymmetric increase in the dispersion of firm growth, which leads to a decline in the skewness. Hence, in this section, we evaluate the response of our modeled economy to a pure change in the variance of firm-level shocks–the typical uncertainty shock considered in the previous literature–and to a change in risk that combines both, an increase in the variance and a decrease in the skewness of firm-level shocks. This is displayed in figure 2.10, where we plot the evolution of several economic aggregates after a shock that combines variance and skewness (blue line with circles), a pure skewness shock (black line with diamond), and pure variance shock (red line with triangles).

[23]Adding labor adjustment costs will trigger an automatic response of labor to changes in risk, increasing the aggregate impact of a change in variance and skewness.

Starting with the effects of a pure uncertainty shock, we see that an increase in the variance of idiosyncratic productivity generates an increase in output and consumption. The difference with respect to a skewness shock is mainly due to the Oi (1961), Hartman (1972), and Abel (1983) effect: a symmetric increase in dispersion pushes up the productivity of some firms at the top of the distribution, which, in the absence of labor adjustment costs, increases labor demand and output for these firms.[24] This increase in productivity of firms at the top of the distribution more than compensates the decrease in productivity and labor demand from firms at the bottom, increasing aggregate output.

Capital investment responds negatively to an uncertainty shock in the first period (top right panel of figure 2.10) first, because of a real-options effect generated by the fixed adjustment cost, and second, because of a change in the composition of assets in the economy as the accumulation of risk free assets increases. After the first few periods, however, these two reverse as investment jumps and the accumulation of risk-free assets declines. In contrast, a pure skewness shock does not generate an increase in aggregate output: a decrease in the skewness implies that the left tail of the productivity distribution widens, generating a decrease in investment due to the real-options effect, an increase in precautionary savings in the risk-free asset, and a muted Oi-Hartman-Abel effect as the right tail of the productivity distribution does not change.

The overall effect of an increase in risk that combines a variance and a skewness shock depends on the relative strengths the real-options channel, the risk-aversion, and the Oi-Harman-Abel effect. For our model to match the asymmetric increase in the dispersion of the sales growth distribution that we observe in the data, an increase in dispersion is mostly due to widening of the left tail of the distribution of firm-level shocks without a parallel widening of the right tail, which commands a decline in the skewness in productivity and sales growth.[25] The combined effect of dispersion and skewness is followed by a significant

[24] We omit the evolution of labor in figure 2.10 since it follows the same pattern of aggregate output.

[25] The asymmetric increase in dispersion generated by the model can be appreciated by comparing the response of the 90th-to-50th and the 50th-to-10th percentiles spreads generated by the model. Appendix figure B.5b displays the evolution of these moments for the three cases we have discussed. In the case of a pure variance shock–red line with triangles–both tails of the distribution expand symmetrically (compare the 50th-to-10th percentile spread to the 90th-to-50th percentiles spread), but in the case of an increase in dispersion that is accompanied by a decrease in the skewness it is only the left tail that expands (measured by the 50th), whereas the dispersion of the right tail almost does not change–blue line with circles. This is exactly what we observe in the data when we compare periods of high and low risk in our model (see table 2.4) and in the data during recessions periods (see figure 2.2).

and persistent decline in aggregate economic activity shown by the blue line with circles in figure 2.10.[26] In this case, output declines almost 2.0% in the first four quarters, the same as consumption, whereas investment drops almost 40% relative to its pre-shock level.

2.5.5 Robustness

In this section, we discuss the robustness of our findings to different parameterizations. Recall that in our baseline results, in the period in which a change in risk occurs, firms do not experience a change in the actual realizations of shocks but only a receive news that in the next period the skewness of productivity shocks, for instance, will be lower. In the next period, however, the firm's productivity distribution changes as the shocks are drawn from a left-skewed distribution. We compare this baseline case to one in which we keep the underlying distribution of firms shocks fixed so that we can evaluate the pure effect of a change in news about the future risk conditions.[27] This exercise is similar to an increase in the probability of a *disaster*, although in our case it represents an increase of *disasters* at the microeconomic level. The blue line with circles in figure 2.11 shows that the overall effect of a skewness shock combines the impact of a change in the perceptions about future risk conditions and the actual change in the realizations of idiosyncratic shocks. In fact, a shock that only represents news about the future risk generates a decline in output of about 0.5%, which is around one-third to the overall decline in our baseline results.

We then study how our results change with the degree of risk aversion of the entrepreneurs and their elasticity of inter-temporal substitution while keeping the rest of the parameters at their values in table 2.6. The red line with triangles in figure 2.11 shows that decreasing entrepreneur's risk aversion, ξ, from 6 to 2, does not impact our main results substantially in terms of aggregate output and consumption, although alters the effect of skewness on the accumulation of capital and the risk-free asset. An increase of the elasticity of intertemporal substitution, $1/\lambda$, from 0.2 to 0.5, does reduce the impact of skewness shocks on output and consumption (green line with squares) although the overall effect is still

[26] It is worth noticing that an aggregate mean shock (a decline in A_t) does not generate a sizable change neither in the dispersion nor the skewness of the distribution of sales growth as we show in figure B.5c.

[27] In particular, we simulate our model using the same realizations of the aggregate risk process used in our baseline analysis. In period T all economies receive a skewness shock, however, in this case, we keep the parameters determining the underlying idiosyncratic productivity process fixed at their pre-shock low-risk level.

significant. Investment, in this case, changes much less relative to the benchmark. These differences highlight the importance of separating the effect of risk-aversion from intertemporal substitution when evaluating the impact of risk shocks.

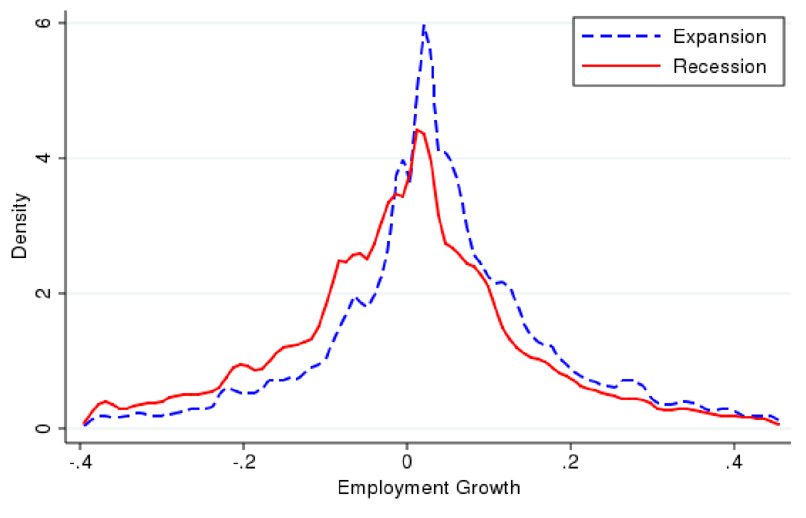
2.6 Conclusions

This paper studies how the distribution of the growth rate of firm-level variables changes over the business cycle. Using firm-level panel for the United States from Census and non-Census datasets, and firm-level panel data for over thirty other countries we reach three main conclusions. First, recessions are characterized by a large drop of the skewness of firm-level outcomes such as employment growth, sales growth, and stock returns. Hence, the skewness of firms' outcomes is strongly procyclical. This decline in the skewness is driven by an uneven change in the dispersion of the distribution of firms' outcomes. In particular, we find that most of the increase observed during the typical recession is accounted for by a left tail that stretches out. Second, the decline in the skewness of firm's outcomes is not only a phenomenon observed in the United States but also in other countries, both developed and developing. Finally, we find strong procyclicality of the skewness at the industry level.

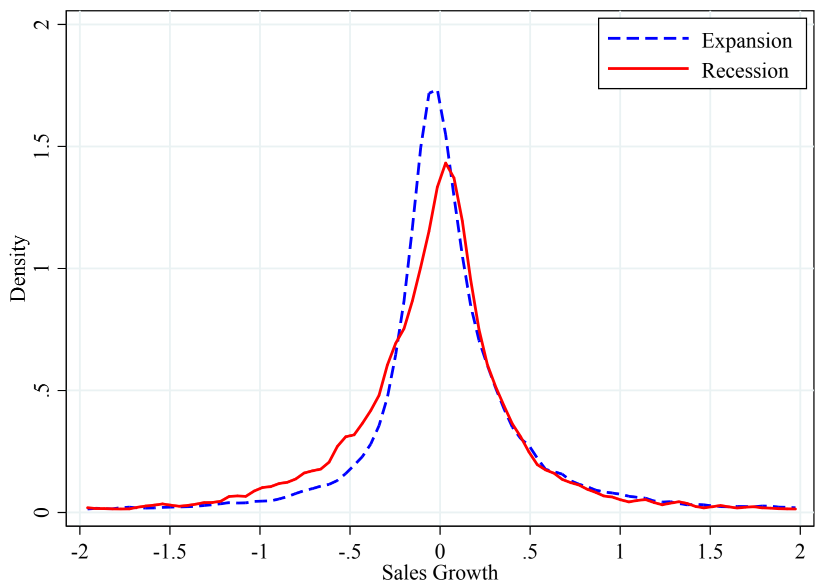
In the second part of our paper, we analyze the impact of a change in the skewness of firms' idiosyncratic productivity in the context of a heterogeneous agents model. We assume that the exogenous idiosyncratic productivity process faced by entrepreneurs is subject to time-varying variance and time-varying skewness and we choose the parameters of this model to match the evolution of the dispersion and the skewness of the sales growth distribution in the United States. Our results suggest that a change in the skewness of the firm-level productivity distribution can by itself generate a significant decline in aggregate economic activity even though the mean and variance of firm's shocks are held constant. In fact, in our modeled economy, a decline in the skewness of firm's shocks of the magnitude observed in the typical US recession generates a drop in GDP of 1.7%. The combined impact of a variance and skewness shock generates an even large decline in output (2.0%), consumption (2.0%), and investment (40.0%).

Figure 2.1: The Skewness of Firm Outcomes Is Lower During Recessions

(a) Census LBD: Employment Growth Distribution



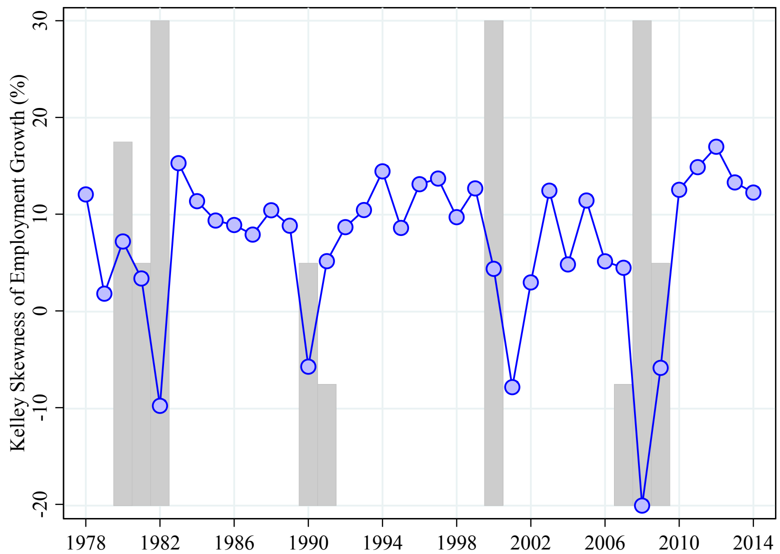
(b) Compustat: Sales Growth Distribution



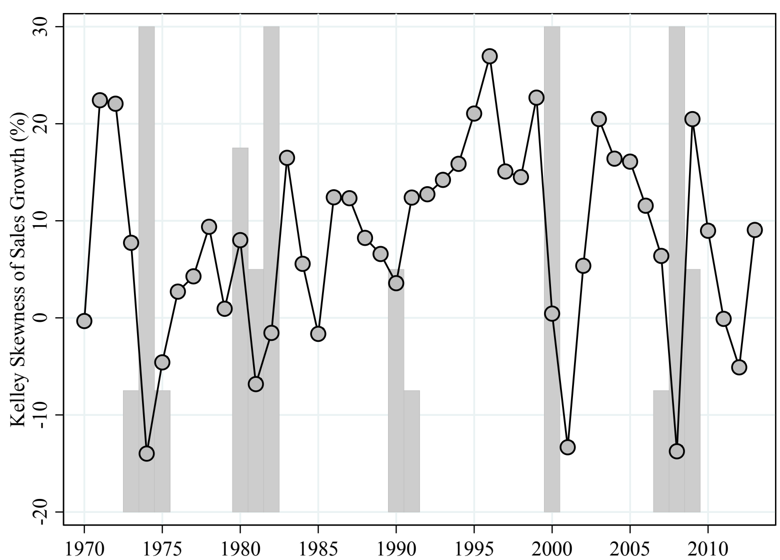
Note: The top panel of figure 2.1 shows the employment-weighted empirical density of the distribution of employment growth for a sample of firms from LBD. The lower panel shows the empirical density of the distribution of sales growth from a sample of publicly traded firms from Compustat. Each density has been rescaled to have a median of zero and unitary variance. The blue-dashed line shows the density of a pooled sample of expansion years (2003 to 2006 and 2010 to 2014) whereas the red-solid line shows the density of a pooled sample of recession years (2001 and 2008). In the top panel, the unscaled 10th percentile of the employment growth distribution during expansion (recession) periods is -16.5% (-26.9%), the 50th is 1.3% (-1.8%), and the 90th is 23.3% (18.0%). In the the bottom panel, the corresponding moments are -21.7% (-47.4%), 5.3% (-3.0%), and 44.6% (33.0%). See appendix B.1 for additional details on the sample construction and moment calculations in the LBD and Compustat.

Figure 2.2: The Skewness of Firm Outcomes is Strongly Procyclical

(a) Census LBD: Skewness of Employment Growth Distribution



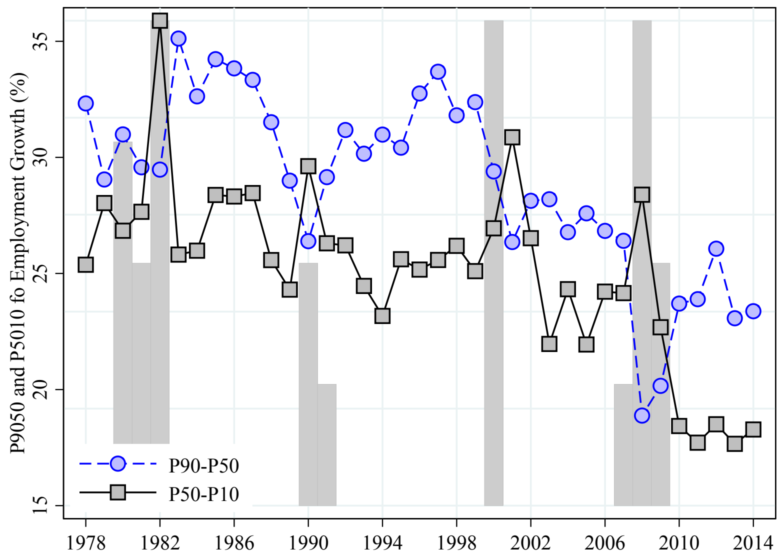
(b) Compustat: Skewness of Sales Growth Distribution



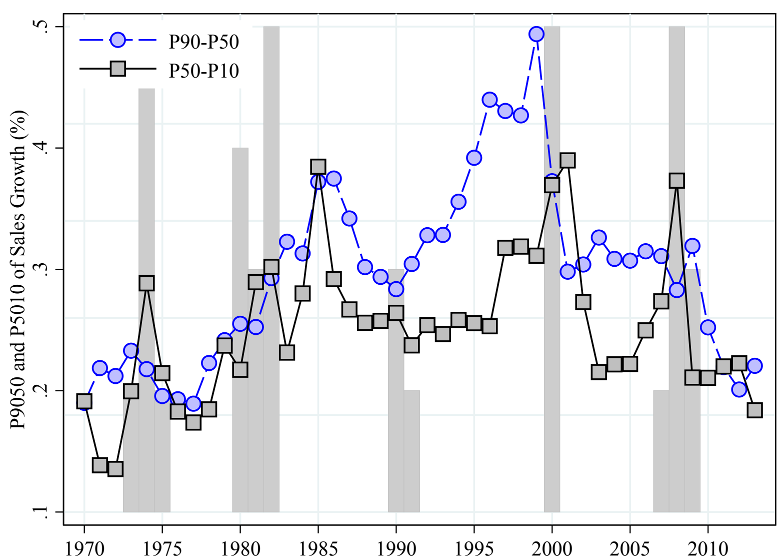
Note: The top panel of figure 2.2 shows the time-series of the cross-sectional Kelley skewness of the distribution of firm employment growth for a sample of firms from LBD. Moments are weighted by the average firm employment between years t and $t+1$. The bottom panel shows the time-series of the cross-sectional Kelley skewness of the distribution of firm sales growth for a sample of publicly traded firms from Compustat. Compustat data shows a large decline in skewness in 2014 which is not found in the rest of the datasets. We are currently investigating the source of this anomaly. The shaded bars represent NBER recession periods. See appendix B.1 for details on the sample construction and moment calculations in the LBD and Compustat.

Figure 2.3: Dispersion of Left Tail of Firms Outcomes is Countercyclical

(a) Census LBD: Dispersion of Employment Growth



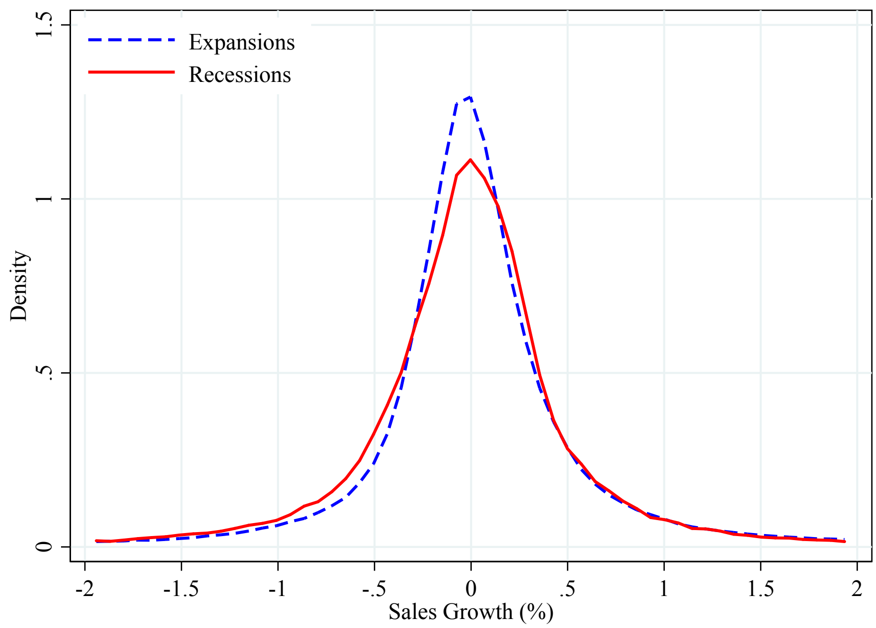
(b) Compustat: Dispersion of Sales Growth



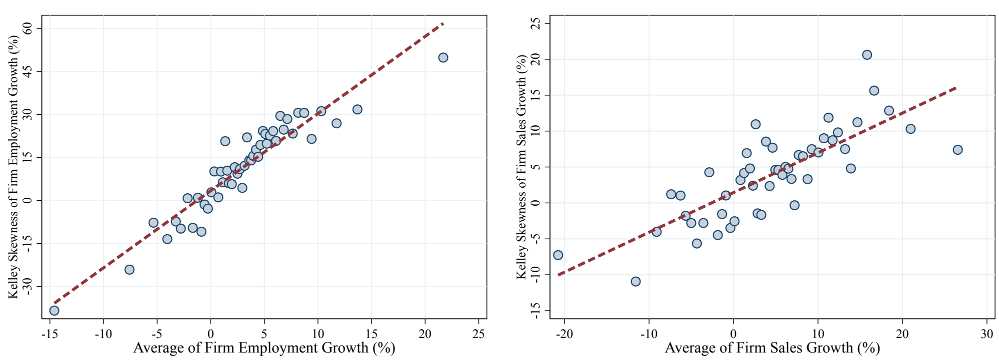
Note: The top panel of figure 2.3 shows the time-series of the cross-sectional dispersion of the distribution of firm employment growth for a sample of firms from LBD. Moments are weighted by the average firm employment between years t and $t+1$. The bottom panel shows the time-series of the cross-sectional dispersion of the distribution of firm sales growth for a sample of publicly traded firms from Compustat. Compustat data shows a large jump in dispersion in 2014 which not found in the rest of the datasets. We are currently investigating the source of this anomaly. The shaded bars represent NBER recession periods. See appendix B.1 for details on the sample construction and moment calculations in the LBD and Compustat.

Figure 2.4: The Skewness of Firm Outcomes is Lower During Country Cycles

(a) Cross-Country: Sales Growth Distribution



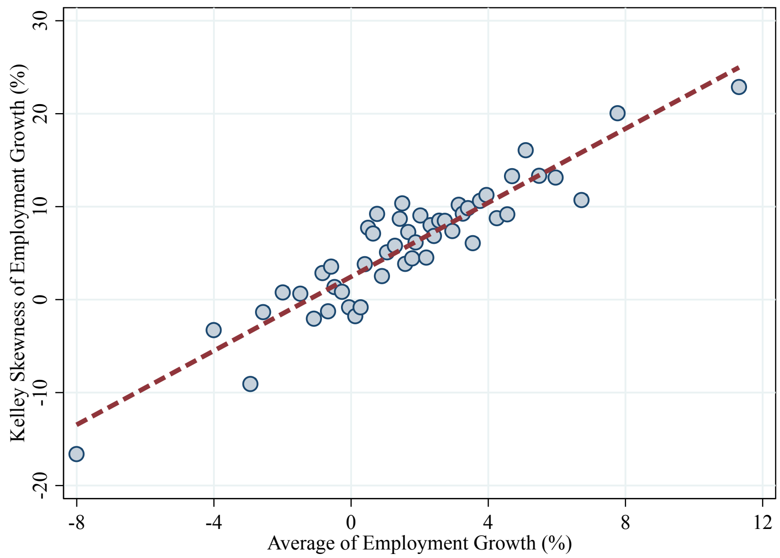
(b) Cross-Country: Firm-Level Employment and Sales Growth



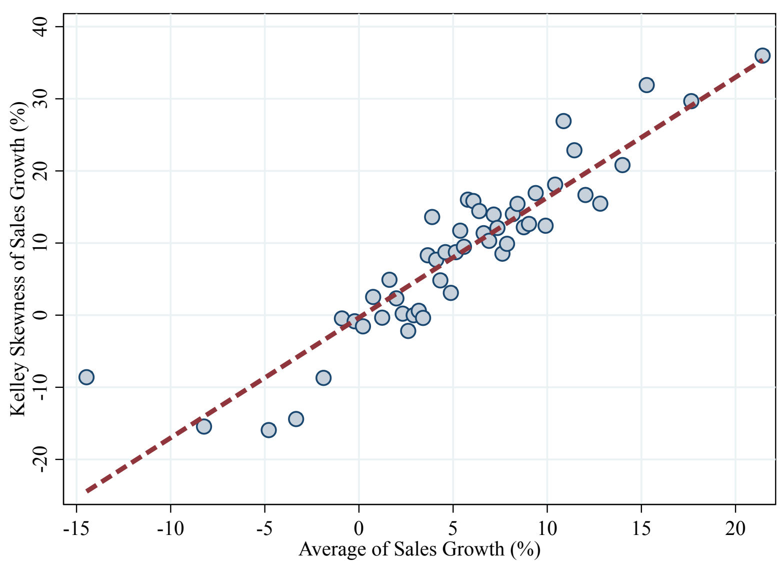
Note: The top panel of figure 2.4 shows the empirical density of the growth rate of annual sales in US dollars for a sample of publicly traded firms from BvD Osiris dataset. Each density has been rescaled to have a median of zero and unitary variance. The red solid line is the empirical density over all the observations of firms during recession years, defined as years in which the country is in the first decile of the country-specific distribution of the growth rate of GDP per capita (74,009 observations). The blue dashed line is the empirical density over all the observations of firms during expansion periods (523,655 observations) which are years not classified as recessions. The unscaled 10th percentile of the sales growth distribution during expansion (recession) periods is -30.5% (-42.4%), the 50th percentile is 5.6% (0.0%), and the 90th percentile is 52.5% (43.6%). The bottom left panel displays a bin scatter plot showing the relation between the within-country average firm employment growth and the within-country Kelley skewness of firm employment growth for a sample of publicly traded firms from BvD Osiris dataset. The bottom right panel shows similar statistics for the within-country firm sales growth distribution. See Appendix B.1 for details on the sample construction and moment calculations.

Figure 2.5: THE SKEWNESS FIRM OUTCOMES IS LOWER DURING INDUSTRY CYCLES

(a) Census LBD: Industry Employment Growth Distribution

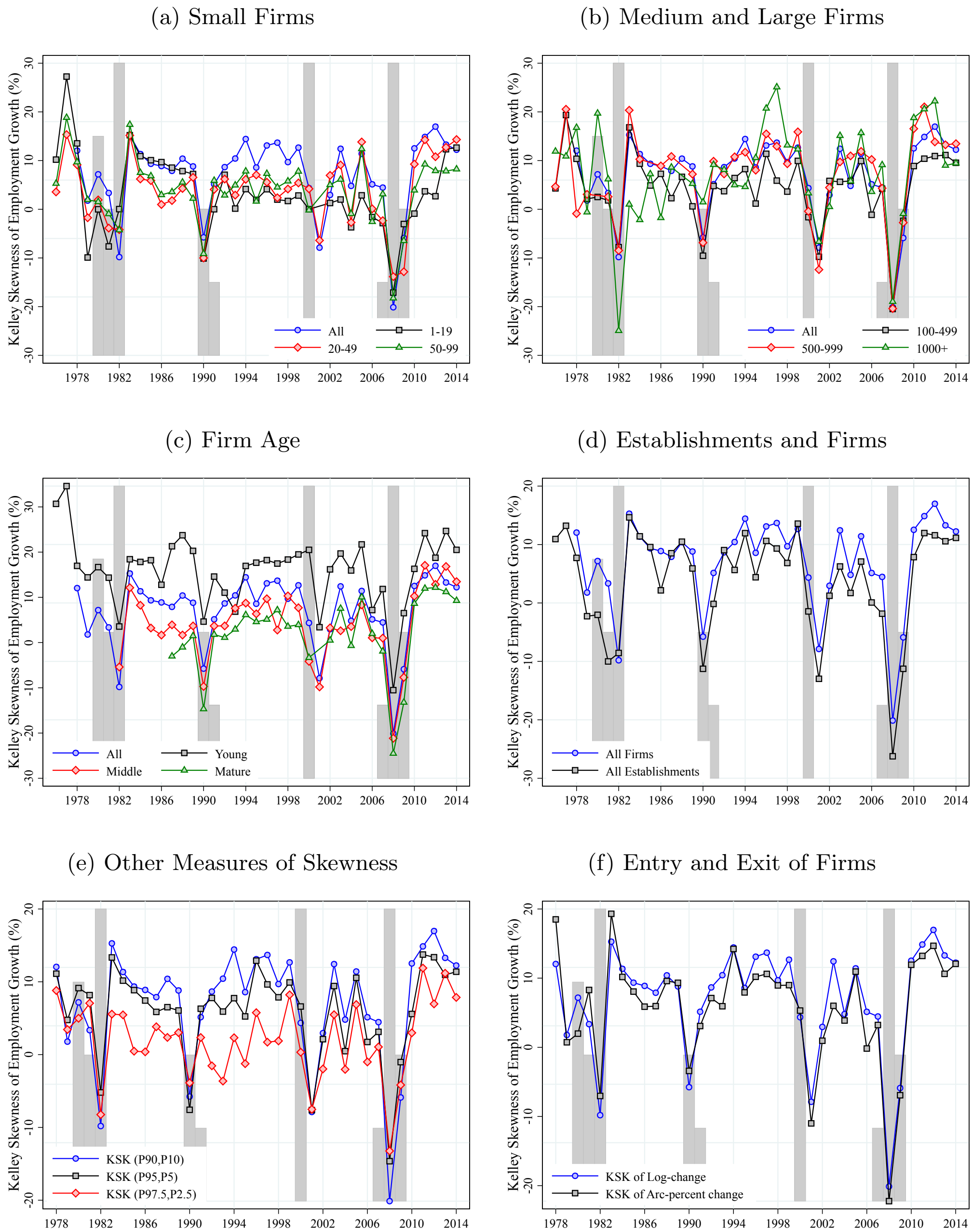


(b) Compustat: Industry Sales Growth Distribution



Note: The top panel of figure 2.5 displays a bin scattered plot showing the relation between the within-industry business cycle, measured by the average growth rate of employment, and the within-industry skewness, measured by the Kelley skewness of firm employment growth for a sample of firms from LBD. Each dot is a quantile of the industry-year distribution of average employment growth where an industry is defined by a 2-digits NAICS group. Moments are weighted by the average firm employment. The bottom panel shows the same statistics for sales growth distribution for a sample of publicly traded firms from Compustat. See Appendix B.1 for details on the sample construction and moment calculations in the LBD and Compustat.

Figure 2.6: Robustness using Census Data



Note: Figure 2.6 is based on a sample of firms from LBD. The top panels show the Kelley skewness of the distribution of firm employment growth within different firm size groups. The center-left panel shows the skewness of the distribution of firm-employment growth within different firm age groups. Young firms are those of less than five years, Middle-aged firms are those between six and ten years old, and Mature firms are those of more than ten years old. Firms already in the sample in 1976 are not considered in any of these groups. Shaded bars represent the share of the year (in quarters) declared as recession years by the NBER. All moments weighted by average employment at the firm or establishment level. See Appendix B.1 for details on the sample construction and moment calculations in the LBD.

Table 2.1: Data and Sample Characteristics

Source	Country	Sample Period	Frequency	Comments
Census	United States	1978-2015	Annual	Employment data for entire nonfarm private sector
Compustat	United States	1970-2017	Quarterly	Employment, Sales, and Stock Prices for publicly traded firms
BvD Osiris	Several countries	1986-2015	Annual	Employment and Sales for publicly traded firms across 44 countries
Global Compustat	Several Countries	1970-2017	Daily	Stock Prices for publicly traded firms across 29 countries

Table 2.2: THE SKEWNESS OF FIRMS OUTCOMES IS LOWER DURING RECESSIONS

Dependent Variable:	Kelley Skewness of the Growth Rate of Firms' Outcomes								
Sample:	United States			Cross-Country			Cross-Industry		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Outcome:	Emp.	Sales	Stock Price	Emp.	Sales	Stock Price	Emp.	Sales	Stock Price
$\Delta GDP_{i,t}$	4.64^{***}	5.37^{***}	2.09^{**}	5.39^{***}	3.19^{***}	2.11^{**}			
	(1.45)	(1.07)	(1.03)	(1.47)	(1.05)	(0.88)			
$\Delta S_{j,t}$							6.62^{***}	13.24^{***}	1.35^{**}
							(1.18)	(1.39)	(0.51)
R^2	0.32	0.23	0.07	0.27	0.38	0.41	0.28	0.40	0.24
N	39	47	184	701	720	2,428	1,045	1,046	4,133
Period	1976-2014	1970-2017	1970-2016	1991-2015	1991-2015	1970-2017	1970-2017	1970-2017	1970-2016
Freq.	Yr	Yr	Qtr	Yr	Yr	Qtr	Yr	Yr	Qtr
F.E.	-	-	-	Yr/Ctry	Yr/Ctry	Qtr/Ctry	Yr/Ind	Yr/Ind	Qtr/Ind
Source	LBD	CSTAT	CSTAT	BvD	BvD	GCSTAT	CSTAT	CSTAT	CSTAT
Sample	-	231K	650K	357K	633K	5,800K	231K	231K	733K

Note: The left panel of table 2.2 shows a set of time-series regressions for the United States in which the dependent variable is the Kelley Skewness of the distribution of one-year firm employment growth for a sample of firms from LBD (column 1), one-year sales growth distribution (column 2), and one-year stock returns (column 3), for a sample of firms from Compustat (CSTAT). In each regression, the independent variable is the annual growth rate of GDP per capita. LBD moments are weighted by firm size measured by the average employment of the firm between years t and $t+1$. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. Columns (4) to (6) show a set of country-panel regressions in which the dependent variable is the Kelley skewness of firm-level outcomes. Employment and sales data come from Bureau Van Dijk's Osiris (BvD) and stock returns are from Global Compustat (GCSTAT). In each regression, the independent variable is the growth rate of annual GDP per capita. Regressions include year and country fixed effects. Columns (7) to (9) show a series of industry-panel regression in which the dependent variable is the Kelley skewness of the within-industry distribution of firm's outcomes. In each regression, the independent variable is the average sales growth within the industry. The raw labeled Sample shows the underlying sample of firms used to calculate the cross-sectional moments. Sample size in LBD not disclosed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.3: Calibrated Parameters

Preferences and Technology		
γ	0.45	Frisch elasticity of labor supply
ψ	2.5	Leisure preference, non entrepreneurs spend 1/3 time working
σ	2.0	Risk aversion, non entrepreneurial sector
$1/\lambda$	1/5	Elasticity of inter-temporal Substitution
ξ	6.0	Risk aversion
β	$0.95^{0.25}$	Annual discount factor of 95%
r	0.005	Annual return of risk-free asset of 2%
α	0.25	CRS production, markup of 33%
ν	0.50	CRS labor share of 2/3, capital share of 1/3
δ	3.8%	Annual depreciation of capital stock fo 14.4%
ρ_a	0.95	Quarterly persistent of aggregate productivity
σ_a	0.75%	Standard deviation of Innovation of aggregate productivity
ρ	0.95	Quarterly persistence of idiosyncratic productivity
Adjustment costs		
ϕ_1	1.50%	Fixed cost of changing capital stock
ϕ_2	7.0	Quadratic cost of changing capital stock
ϕ_3	34.0%	Resale loss of capital

Note: Table 2.3 shows the calibrated parameters referring to preferences, technology, and adjustment costs.

Table 2.4: Risk Process Moments

	$P90-P10$	$P90-P50$	$P50-P10$	KSK	Yrs
Data					
Low Risk	0.54	0.30	0.24	0.10	03-06;10-14
High Risk	0.70	0.31	0.39	-0.11	01,08
$\Delta\,(H-L)$	0.16	0.01	0.15	-0.20	-
Model					
Low Risk	0.48	0.27	0.20	0.15	-
High Risk	0.58	0.26	0.32	-0.10	-
$\Delta\,(H-L)$	0.10	-0.01	0.12	-0.25	-

Note: The top panel of table 2.4 shows cross-sectional moments of the annual growth rate of quarterly sales from Compustat for low risk periods–quarters in the years 2003 to 2006 and quarters in the years 2010 to 2014–and high risk periods–quarters in years 2001 and 2008. Quarters in years 2002 and 2009 are discarded for not representing full recession years. The model moments, shown in the lower panel of table 2.4, are calculated from a 5,000-quarters simulation with the first 500 periods discarded.

Table 2.5: Targeted Moments for Numerical Comparison

	$P9010$	$P9050$	$P5010$	KSK
Low Risk	0.54	0.30	0.24	0.10
High Risk	0.70	0.31	0.39	-0.10
Only Skewness	0.54	0.243	0.297	-0.10
Only Variance	0.70	0.39	0.31	0.10

Note: Table 2.5 shows the target used in the estimation of the firm-level productivity process. Rows labeled “Low Risk” and “High Risk” are used in the baseline estimation. The values for “Only Skewness” are used to estimate the parameters when the economy is shocked with a change in the skewness only. Similarly, the values for “Only Variance” are used to estimate the parameters when the economy is assumed to be shocked only by a change in the variance of firms’ shocks while keeping the skewness constant.

Table 2.6: Parameters of the Stochastic Process

Parameter of Idiosyncratic Stochastic Process		
σ_1^L	1.45	Standard deviation of first mixture in low risk periods (%)
σ_2^L	7.55	Standard deviation of second mixture in low risk periods (%)
μ^L	-0.92	Mean of first mixture in low risk periods (%)
p^L	63.67	Probability of first mixture in low risk periods (%)
σ_1^H	4.37	Standard deviation of first mixture in high risk periods (%)
σ_2^H	9.06	Standard deviation of second mixture in high risk periods (%)
μ^H	1.98	Mean of first mixture in high risk periods (%)
p^H	78.28	Probability of first mixture in high risk periods (%)
Transition Probabilities of Risk States		
π_L	0.97	Quarterly probability of remaining in low risk state
π_H	0.84	Quarterly probability of remaining in high risk state

Note: The top panel of table 2.6 shows the parameters of the stochastic process of firm-level productivity. We target moments of the annual change of quarterly sales in Compustat. The parameters for low-risk periods (denoted by an upper script L) are obtained by targeting the P90-P10, P90-P50, P50-P10, and Kelley Skewness of the sales growth distribution for the all the full expansion years between 2000 and 2014. The parameters for high-risk periods (denoted by an upper script H) are obtained by targeting the same set of moments for years 2001 and 2008 (full recession years). The transition probability π_L is calculated as the share of expansion quarters that were followed by another expansion quarter whereas π_H is calculated as the share of recession quarters that were followed by another recession quarter using data from 1970 to 2014.

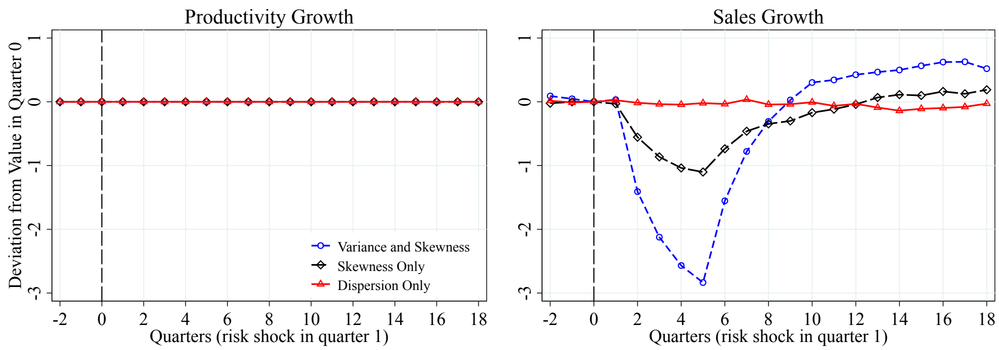
Table 2.7: Business Cycle Statistics

	Data			Model		
	$\sigma(x)$	$\sigma(y)/\sigma(x)$	$\rho(x,y)$	$\sigma(x)$	$\sigma(y)/\sigma(x)$	$\rho(x,y)$
Output	1.47	1.00	1.00	2.00	1.00	1.00
Capital Investment	6.86	4.64	0.91	9.38	4.69	0.30
Consumption	1.21	0.82	0.87	1.81	0.91	0.65
Hours	1.89	1.28	0.87	2.00	1.00	1.00

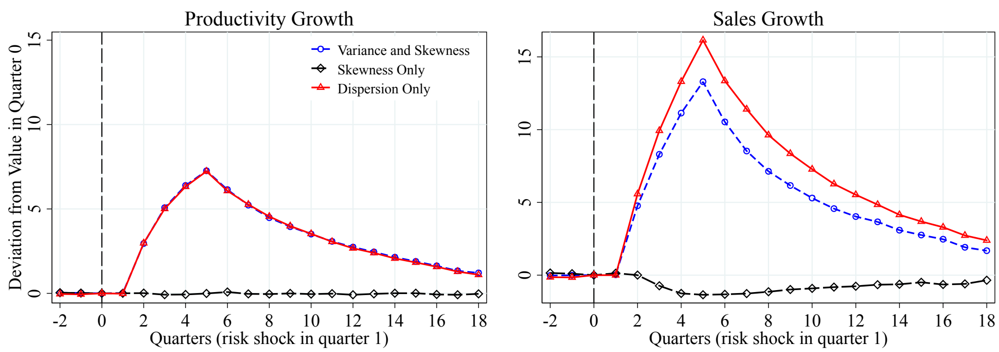
Note: The left panel of table 2.7 displays busyness cycles statistics for quarterly US data covering 1970Q1 to 2017Q4. The column $\sigma(x)$ is the standard deviation of the log variable in the first column. The column $\sigma(y)/\sigma(x)$ is the standard deviation of the variable relative to the standard deviation of log output. All business cycle data are current as of February 03, 2019. Output is real gross domestic product (FRED GDPC1), investment is real gross private domestic investment (FRED GPDIC1), consumption is real personal consumption expenditures (FRED PCECC96), and hours is total non-farm business sector hours (FRED HOANBS). The second panel contains business cycle statistics computed from a simulation of the model of 5000-quarter with the first 500 periods discarded. All series are HP-filtered with smoothing parameter 1,600, in logs expressed as percentages.

Figure 2.7: Productivity and Sales Growth after an Increase in Risk

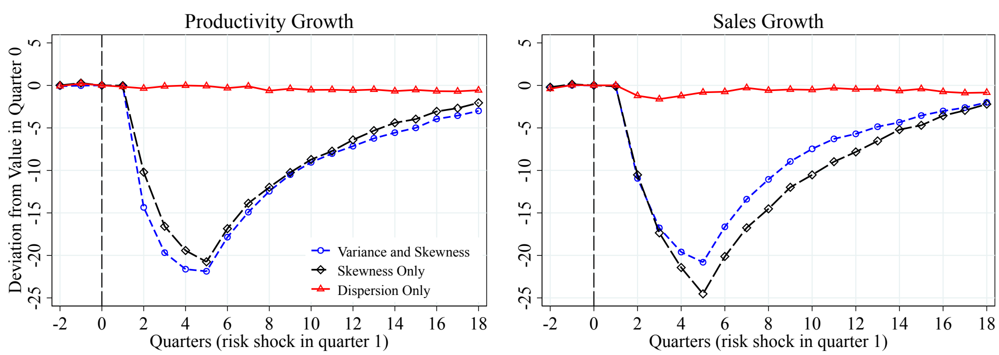
(a) Average



(b) P90-P10

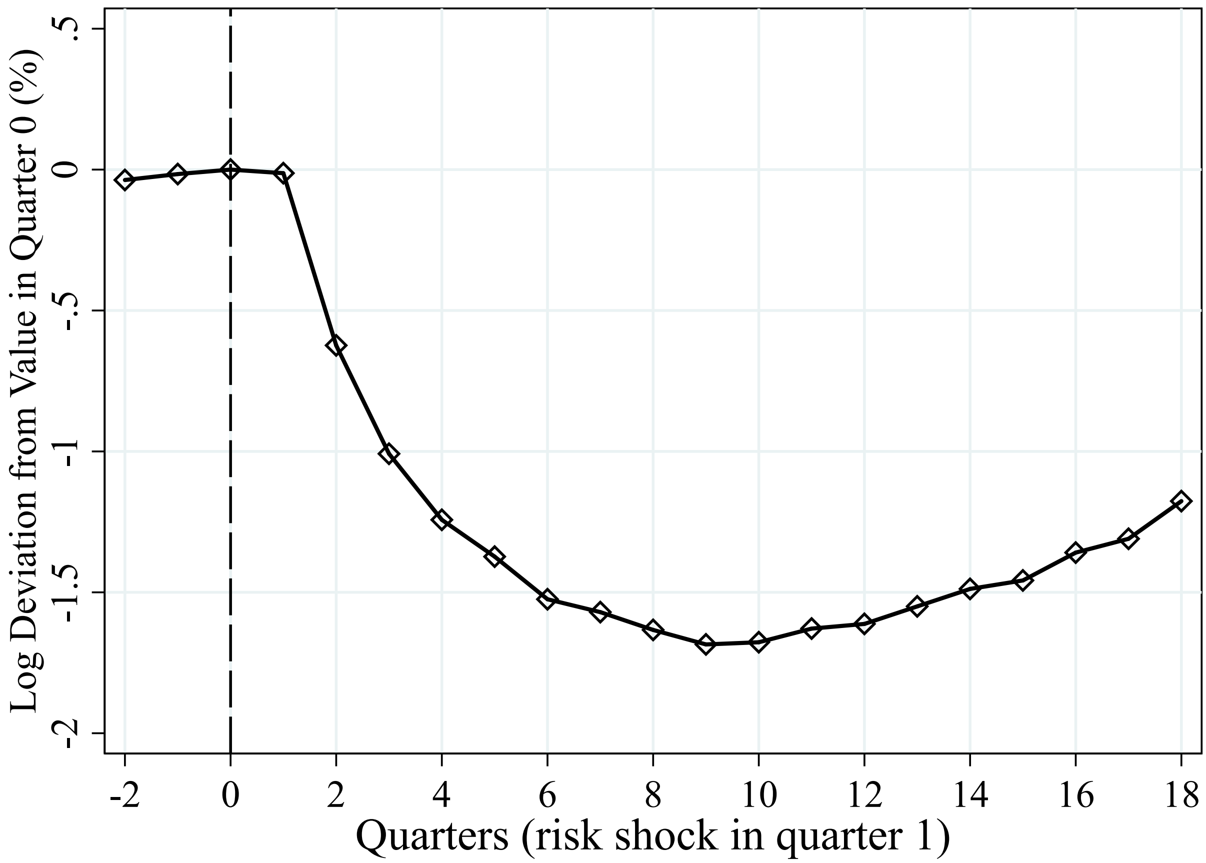


(c) Kelley Skewness



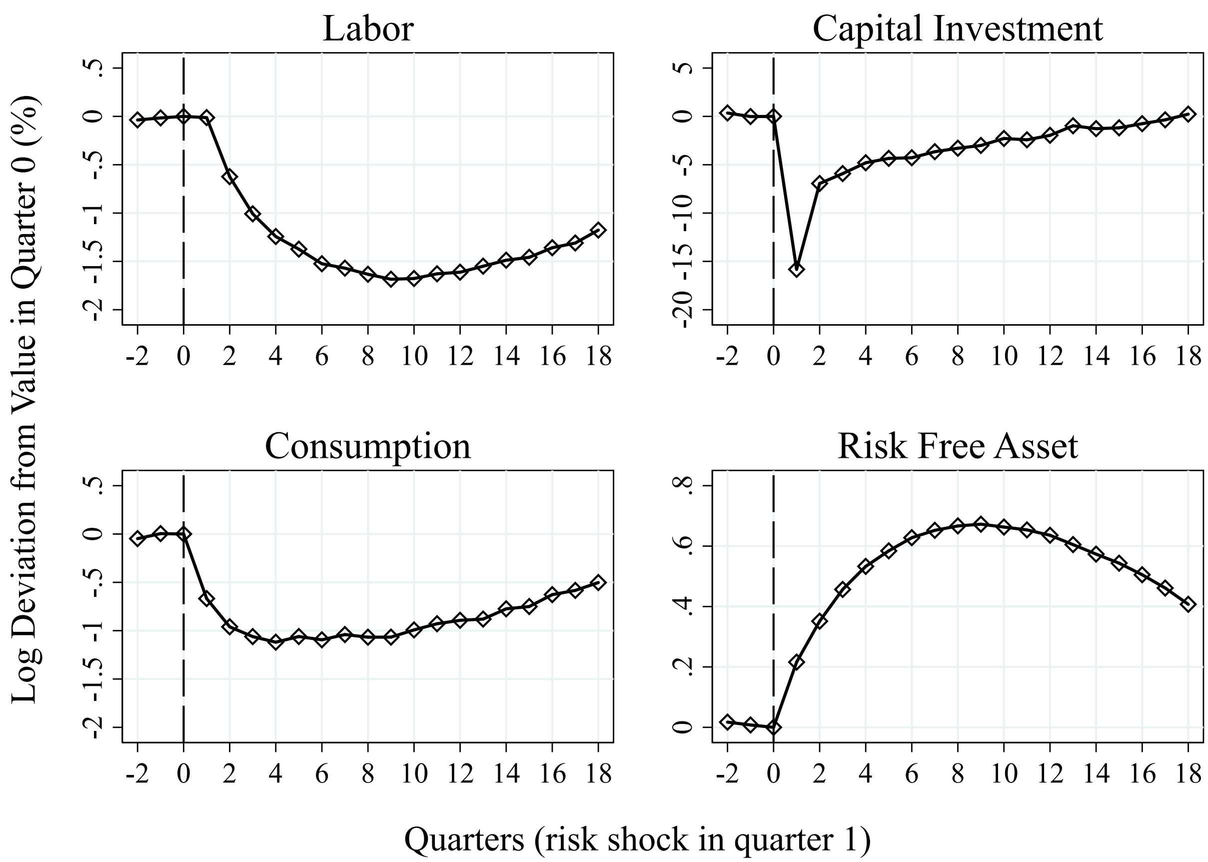
Note: The top-left panel of figure 2.7 shows the average of the one-year productivity growth distribution ($\Delta e_{j,t} = e_{j,t} - e_{j,t-4}$) whereas the top-right shows the average of the one-year sales growth distribution ($\Delta y_{j,t} = \log y_{j,t} - \log y_{j,t-4}$) for different risk shocks. The middle and bottom panels show the dispersion and skewness. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterward. We plot the deviation relative to the moment value in quarter 0.

Figure 2.8: Effect of Skewness Shock in Output



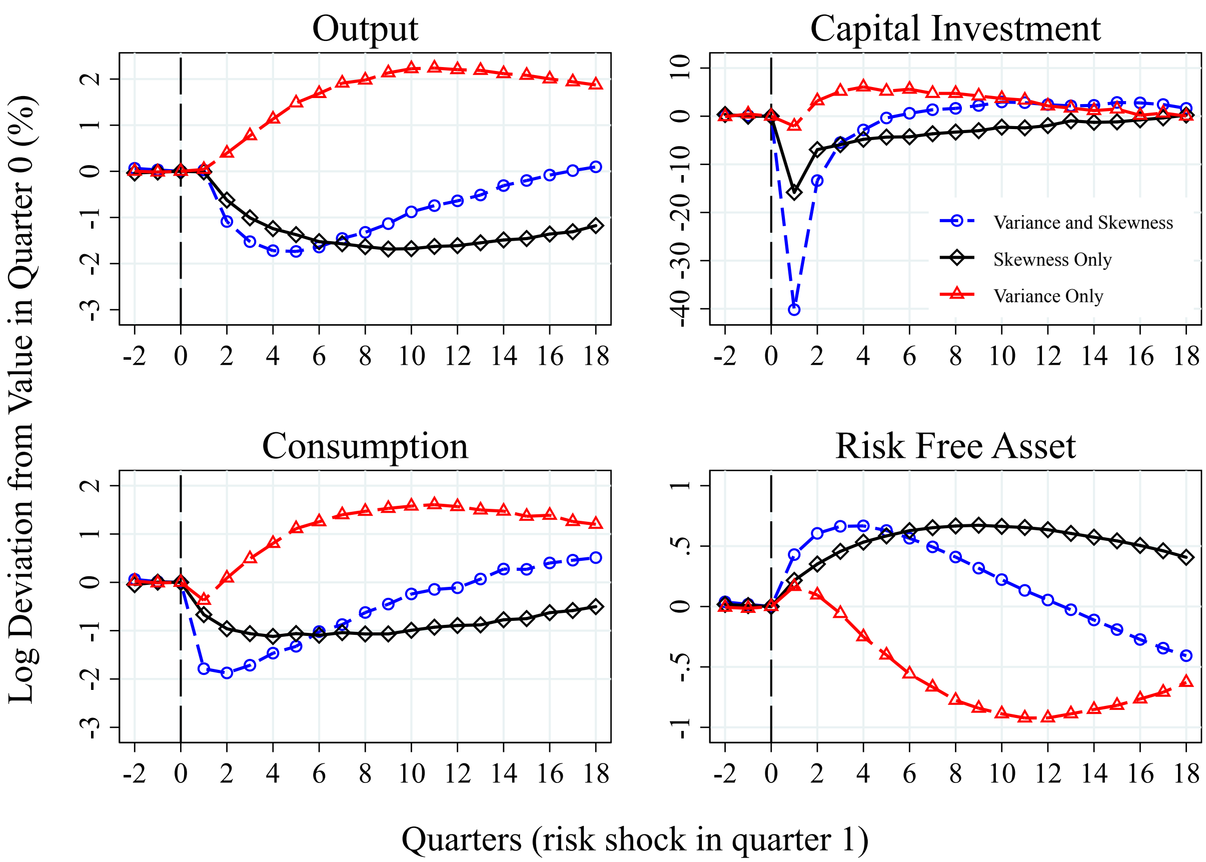
Note: Figure 2.8 shows the effect of a decline in the skewness of firm idiosyncratic productivity. The plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of Output from its value in quarter 0.

Figure 2.9: Effect of Skewness Shock on Macro Aggregates



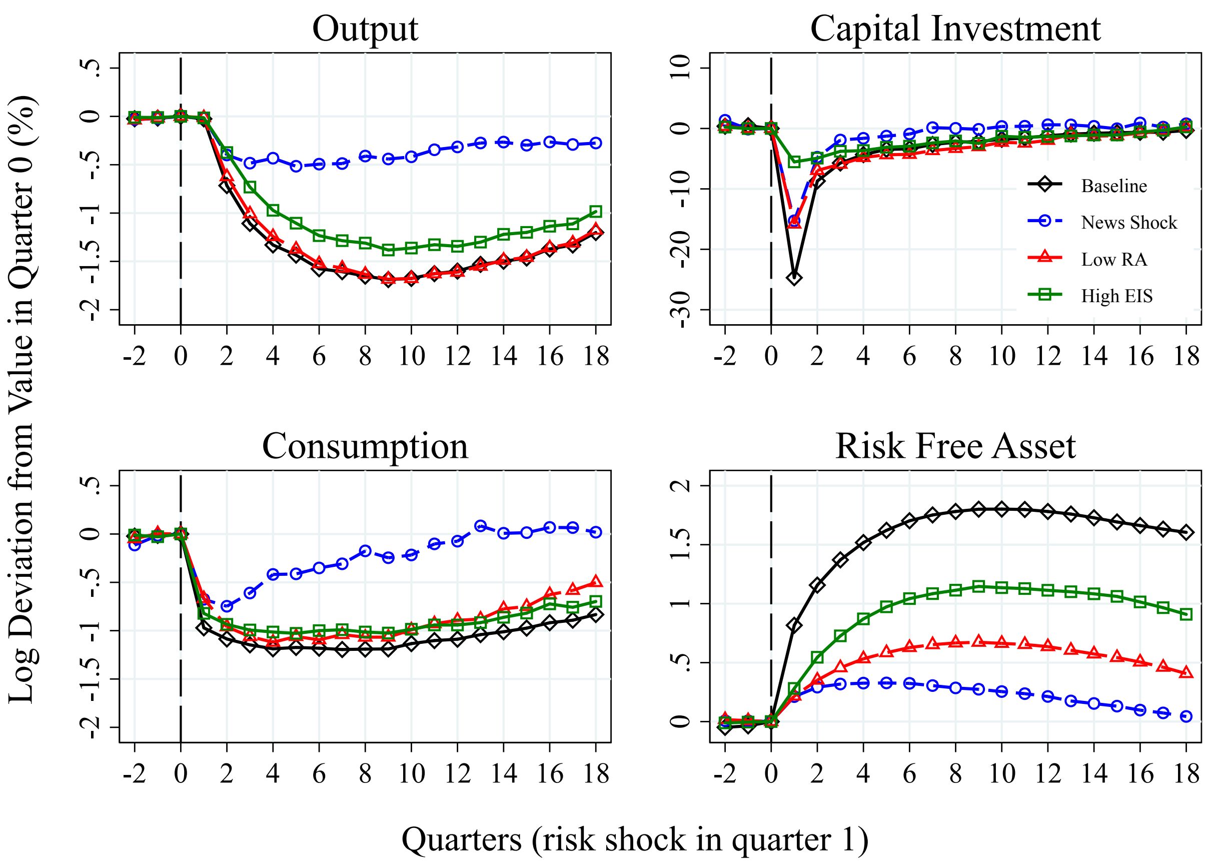
Note: Figure 2.9 shows the effect of a decline in the skewness of firm idiosyncratic productivity. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of each macroeconomic aggregate from its value in quarter 0.

Figure 2.10: Effect of Skewness and variance Shock on Macro Aggregates



Note: Figure 2.10 shows the effect of a decline in skewness of idiosyncratic shocks (black line with diamonds), an increase in variance of idiosyncratic shocks (red line with squares), and a decrease in skewness paired with an increase in the variance of idiosyncratic shocks (blue line with circles) for different macroeconomic outcomes. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a drop in the skewness, increase in variance, or both, in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of each macroeconomic aggregate from its value in quarter 0. Labor is omitted since it follows the same pattern of Output.

Figure 2.11: Effect of Skewness Shocks under Different Parameterization



Note: Figure 2.11 shows the effect of a skewness shock (black line with diamonds) and the effect of a shock that only represents news about the future conditions of skewness without a change in the realizations of the idiosyncratic shocks. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a drop in the skewness in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of each macroeconomic aggregate from its value in quarter 0. Labor is omitted since it follows the same pattern of Output.

Chapter 3

Heterogeneous Passthrough from TFP to Wages

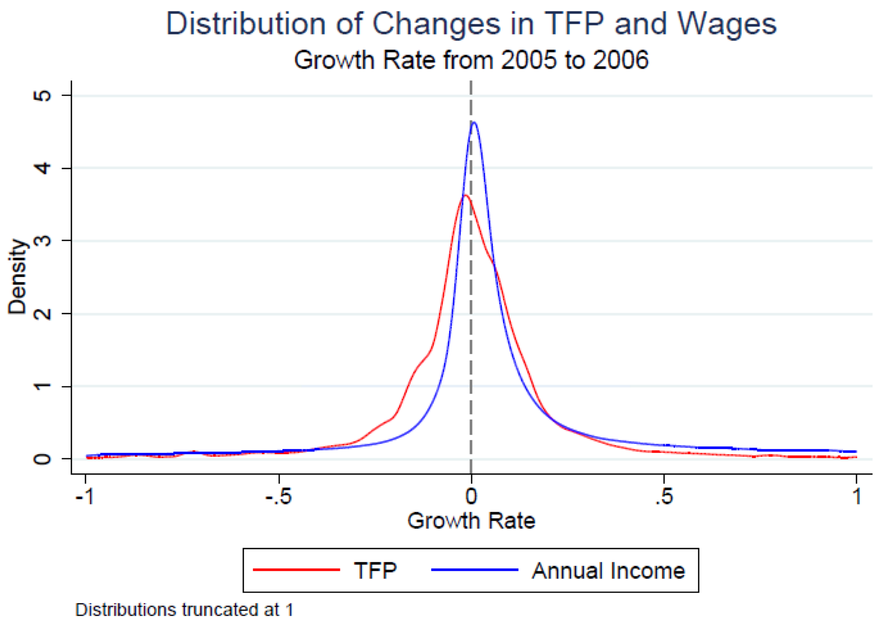
3.1 Introduction

What is the role of firm productivity shocks in workers' income instability?[1] To answer this question we study the the impact of firms' shocks on workers' wages, which we define as passthrough. More precisely, we use matched employer-employee data from Denmark to measure passthrough as the elasticity of workers' wages relative to firms' productivity shocks. We then carefully study how passthrough varies across firms and workers of different characteristics and over time.

To illustrate the large amount of heterogeneity present in firm productivity and labor earnings growth, figure 3.1 shows the density of workers' wage growth and firms' Total Factor Productivity (TFP) growth. Both distributions show significant dispersion and wide tails, indicating large swings in wages and productivity. Importantly, the changes in firms' TFP and the changes in workers' wages are correlated. Figure 3.2 displays the average and standard deviation of wage growth for each percentile of the TFP growth

[1] With Mons Chan and Ming Xu. We thank Fatih Guvenen, Sergio Ocampo, and seminar participants at the 2nd Dale T. Mortensen Centre Conference, SEA 2018 meetings, Queen's University, and the University of Minnesota for helpful comments and discussions. We also thank the Department of Economics and Business at Aarhus University for support and making the data available.

Figure 3.1: Distribution of Changes in TFP and Wages



Note: The red line in figure 3.1 shows the distribution of one-year log change in firm-level Total Factor Productivity (TFP) calculated for a sample of Danish firms that have more than one employee in 2005 and 2006. The blue line shows the distribution of the log change of real annual earnings for a sample of Danish workers between 2005 and 2006. See section 3.2 for additional details in the sample selection.

distribution.[2] We observe four important aspects. First, the average wage growth across the TFP growth distribution is mostly flat, indicating that firms do not completely adjust worker's wages in response to idiosyncratic shocks. Second, there is positive wage growth even among firms experiencing negative changes in productivity (those at the bottom decile of the TFP growth distribution). Third, workers at high growth firms (those at the top decile of the TFP growth distribution) experience wage growth that is three times higher on average than the wage growth experienced by workers at low growth firms. Finally, the right panel of figure 3.2 shows that individuals who work at high or low TFP growth firms experience almost twice as much wage growth dispersion in a given year relative to individuals working in firms in the middle of the TFP growth distribution.

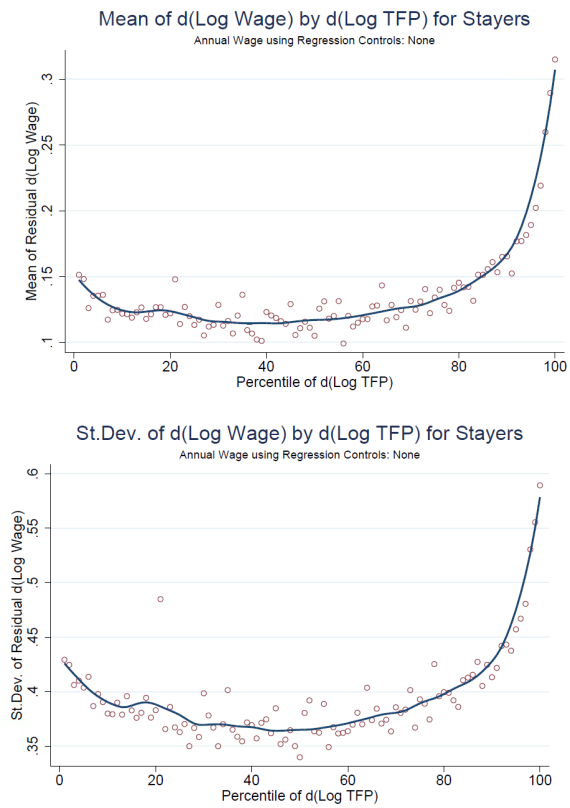
[2]Details on the sample selection as well as the calculation of labor income and firm-level productivity are discussed in section 3.2 and section 3.3.

Our paper provides four contributions to the empirical passthrough literature. First, we use individual-level panel data on every worker in Denmark to study the effects of firm-level shocks separately for stayers (workers that remain in the firm), switchers (workers that change firms), and those transitioning in and out of unemployment. This enables us to estimate the effect of productivity shocks on wages separately from the effects of job separation. Second, the existing literature has focused almost exclusively on continuing workers and ignores the endogenous selection of firms and workers. Endogenous selection on the workers' side can arise, for instance, when workers, who would have experienced a large wage decline from when their firm experiences a negative shock, switch firms to avoid a drop in their wages. Similarly, a firm that normally passes productivity shocks to wages may go out of business after a large productivity drop, reducing the measured passthrough on continuing workers. Ignoring this selection problem could underestimate the passthrough and overstate firms' insurance against shocks. In this paper, we address these concerns by using exogenous variation derived from spousal linkages. Third, we use the richness of our dataset to provide a direct measure of firm-level TFP for the entire private sector of the Danish economy. Our methodology allows us to separately study the effects of persistent versus transitory shocks to firm TFP as well as asymmetric passthrough from TFP to wages. This is a significant departure from the existing literature which uses indirect measures of productivity such as value-added, revenues, or output per worker.[3] Fourth, we exploit the breadth of our dataset to study how the passthrough from firm shocks to worker wages varies across narrow population groups defined simultaneously by firm characteristics (industry, size, productivity level, etc.) and worker characteristics (age, education, income level, tenure, etc.), and over the business cycle.

Our novel approach for controlling for selection bias exploits linked spousal information. In particular, we use variation in marriage status, spousal employment decisions and income shocks to estimate the probability that individual moves across firms. Our identification strategy rests on the assumption that changes in a spouse's income, employment status, or spouse's firm productivity, has a significant impact on the decision of an individual to stay or not in a particular job, but such changes are uncorrelated with the worker's

[3]This mostly due to data limitations. For instance, Juhn et al. (2018) use the change in revenues as the firm-level shock affecting workers' earnings and Guiso et al. (2005) use the residuals of a regression of value added on firm-level observables.

Figure 3.2: Wage Growth Across the TFP Growth Distribution



Note: The left (right) panel of figure 3.2 shows the average of (standard deviation of) the wage growth within each percentile of the firm-level TFP growth distribution for a sample of Danish workers and firms. See section 3.2 for additional details in the sample selection.

within-firm wage growth or the productivity shocks affecting the firm where the individual works (unless both spouses work at the same firm, which we excluded in our data). We find that controlling for selection greatly increases the estimated passthrough from firms' productivity shocks to workers' wages.

Overall we find large and economically significant passthrough from firm shocks to wages. In particular, after we have controlled for selection, we find that an individual who works at a firm which experiences an increase of TFP of one standard deviation receives an increase in annual earnings of \$1,500, which is around 3% of the Danish income per capita. Considering that in any given year 33% of firms and 40% of workers in Denmark experience

a TFP shock of at least one standard deviation away from the average, the effect of firm-level shocks on wages is quite substantial not only at the micro- but also at the macro-level. Furthermore, and in contrast to most of the previous literature, we find that persistent and transitory shocks to firm's productivity are passed in equal magnitude to workers wages.
The effect of TFP changes on workers that transition across firms has been largely ignored by the literature and to the best of our knowledge, this is the first paper to separately analyze the impact of productivity shocks on the wage of workers that switch across firms. Analyzing this group is important as they represent a large fraction of the workforce: in any given year around 20% of full-time Danish workers change employer. We find that the effect of between-firm productivity differences on the wages of individuals that move across firms is large and of greater magnitude than the effect of within-firm TFP shocks on the wages of stayers. In particular, a worker that moves across firms whose TFP differ by one standard deviation experiences an income change of $5,200.[4]
We use the richness of our dataset to analyze how the passthrough differs across firm characteristics, workers characteristics, and over the business cycle. First, we analyze the differential effect of productivity shocks for workers at high and low ranks of the income distribution. We find that high wage workers (those in the 5th quintile of the income distribution) are much less insulated from changes in firm productivity than low wage workers (those in the 1st quintile of the income distribution). In fact, we find that the passthrough for high wage workers is three to four times larger than the passthrough for low wage workers. Young workers (workers of 35 years or less) are also more exposed to firms productivity shocks than older workers (workers of 50 years or more). This is both because young workers work in firms that pass a larger proportion of shocks to wages and because the volatility of productivity shocks of the firms where young people work is higher. We also find substantial heterogeneity in passthrough for firms in different industries after correcting for selection. For instance, the passthrough for workers in the Transportation or Hospitality sectors is two to three times larger than for workers in ICT or Finance. This is surprising considering that Finance has a larger fraction of workers under performance pay schemes that typically tie workers' income to firms' outcomes.
Motivated by the robust empirical evidence that firm productivity shocks have a significant

[4]Note that the TFP change for switchers is the difference in productivity between two different firms rather than the within-firm shock to productivity.

impact on workers wages, in the second part of our paper we estimate a flexible stochastic income process that captures the salient features of the relation between firm-level shocks and the passthrough to workers' wages. In particular, we consider an income process similar to Low et al. (2010) modified to take into account the rich heterogeneity in passthrough observed in the data. To capture the marked asymmetry of passthrough between positive and negative shocks, our preferred specification considers a passthrough coefficient of firm's productivity to wages that is different depending on the sign of the productivity change. Our estimation, which is carried out using indirect inference, suggests that firms have a large role in determining income growth dispersion and income inequality.

In the final part of our paper, we embed the estimated stochastic process into a life-cycle consumption savings model with incomplete markets. This framework allows us to calculate the welfare and distributional implications of the partial and heterogeneous passthrough we document in this paper. Our model does a good job in accounting for the extent of income and wealth inequality we observe in Denmark. In our main quantitative exercise, we ask how much value the workers in this economy, in terms of lifetime consumption, assign to the insurance provided by firms. We do so by comparing our benchmark economy to an economy in which firms' shocks are fully passed to workers wages (passthrough equal to one). Our preliminary results suggest that the insurance provided by firms is of little value for workers. This is because an increase in the passthrough has two opposite effects on welfare. On the one hand, higher passthrough has a negative impact as it increases earnings instability. On the other hand, because a larger fraction of the positive shocks is passed to workers their average wage increases. The overall effect depends on the ability of workers to insure against the increase in income risk. In our current steady-state comparison with infinitely lived workers with access to a risk-free asset, an individual has enough time to offset the negative impact of an increase in wage dispersion by increasing savings. In other words, a risk free asset provides enough flexibility to workers to compensate for the decline in the insurance provided by firms by increasing capital accumulation. At the same time, higher average permanent income reduces the necessity of workers to save. These two offsetting effects imply a muted impact on welfare from an increase in the passthrough of firms' shocks to workers' wages and a small increase in capital in the economy. Our ongoing work aims to fully account for the life cycle income profile and have a more realistic asset market that resembles the frictions in the financial market faced by workers, both of which will likely increase the value of the insurance provided by firms.

Our paper relates to several strands of the literature. First and foremost, we contribute to the literature that studies the relationship between firm shocks and worker earnings. Guiso et al. (2005) analyze the degree of insurance provided by firms using matched employee-employer data from Italy. Their paper, however, does not analyze how firm-level productivity affects employment transitions which might explain a large fraction of the earnings instability observed in the data. Barth et al. (2016) and Juhn et al. (2018) also study the heterogeneity of passthrough from firm's shocks to wages. Barth et al. (2016) report that almost three quarters of the dispersion in wage levels is accounted for by differences in TFP levels across firms whereas worker characteristics contribute little. Bagger et al. (2014) use Danish data to study the importance of firm level productivity for wage dispersion, the role of rent sharing between workers and firms, and labor force composition within the firm. They document an important role for fixed TFP differences across firms in the determination of earnings level dispersion. These papers, however, do not analyze the role of firm-level TFP shocks for the dispersion of earnings growth and do not take into account the effects of firm level shocks on employment transitions. These paper also do not address the selection issue which is at the center of our paper.[5]

Our paper also contributes to the understanding of labor income risk. Since the seminal work of Gottschalk et al. (1994), several papers have studied the extent of labor income instability and its evolution over time.[6] Due to data limitations, however, most of the papers in the literature do not consider the role of firms in driving labor income instability. An exception is the work of Comin et al. (2009) that studies whether firm's revenue volatility is passed to average wage instability using data from a panel of publicly traded firms and worker level information from survey-level data. The authors find a positive relation between firms and worker wage volatility. Relative to this paper, we use an employer-employee matched administrative data set that allows us to have a tighter link between firm shocks and earnings instability.

[5]Several recent papers study the relation between firm's shocks and worker's wages. See for instance Lamadon et al. (2017), Friedrich et al. (2014),Carlsson et al. (2015), Garin et al. (2018), Guertzgen (2014), Kline et al. (2018),Rute Cardoso and Portela (2009), Lagakos and Ordonez (2011), among others.

[6]See for instance Sabelhaus and Song (2009), Sabelhaus and Song (2010), Ziliak et al. (2011), and Guvenen et al. (2014).

The rest of the paper unfolds as follows: In section 3.2, we introduce our data source and sample selection. We then discuss our estimation strategy for various specifications of analysis in section 3.3. The baseline model and results with and without selection correction are shown in section 3.4. We then proceed to explore various dimensions of heterogeneity along workers, firms and the timing dimension in section 3.5. Section 3.6 presents our quantitative analysis. Section 3.7 concludes our work.

3.2 Data

Our main source of information is a matched employer-employee administrative dataset from Statistics Denmark. We combine several large panel datasets for our analysis. Worker data comes from the Integrated Database for Labor Market Research which contains employment and personal information for the entire population of Denmark. In particular, we observe annual wages, hourly wages, number of days worked, occupation, labor market status, position within the firm, age, gender, education, and tenure. Crucially, this dataset identifies the firm in which each worker was employed at November of each year. We also have spousal links, which means we observe the same information for everyone's spouse across time. This spousal information will be crucial when estimating the first-stage selection model we use in 3.3.2 to correct for selection bias in the passthrough estimation. Our main measure of labor income of an individual is equal to the worker's hourly wage times the total number of hours she would have worked in a year as a full time worker. In this way we avoid our results being influenced by changes in the number of hours that individuals work in a year. We then consider full time workers who are 15 years and older, whose annualized earnings is above 30,000 DKK (about $4600 USD), and who are not working in the public sector or are self-employed.

We match this individual-level panel to a firm-level panel, the Firm Statistics Register, which contains accounting and input use data for the universe of Danish firms. The key variables we use are firm annual revenues, value added, capital stock, intermediate expenditure and employment, as well as firm age, location, and industry. This data allows us to construct robust measures of TFP following the methods developed by Levinsohn and Petrin (2003), Ackerberg et al. (2015), Gandhi et al. (2018), and others. We also link in firm-product data on physical sales, market shares, input and output prices, and

Table 3.1: Summary Statistics

	Mean	**Std.dev.**	**N**
	Workers Characteristics		
Annual Wages (dkk)	363,661	208,240	8.98M
Hourly Wages (dkk)	234	147	7.36M
Age	41.7	11.3	8.98M
	Firms Characteristics		
Log Value Added	14.6	1.33	0.71M
Log TFP	7.94	0.58	0.71M
Firm Age (years)	13.1	12.5	0.71M
Number of Employees	19.8	192.7	0.71M

1 US$= 6.55 dkk

Note: Table 3.1 shows summary statistics for the sample used in our analysis. Monetary values are expressed in Danish kroner.

imports/exports. This latter data allows us to construct exogenous firm-level TFP shifters so that we can explore the causal relationship between TFP shocks and wage growth. For our baseline analysis, we keep all firms in the private sector with at least one employee. We restrict our analysis to the period from 1995 to 2012. Our sample selection leaves us with about 8.98 million worker-year observations for our primary empirical analysis and 0.71 million firm-year observations. Basic summary statistics can be found in Table 3.1.[7]

3.3 Estimation Strategy

In this section we discuss our estimation strategy. Section 3.3.1 describes our baseline regression model. Section 3.3.3 provides the details of our TFP estimation. Section 3.3.2 discusses our strategy for dealing with the potential bias of the basic model in section 3.3.1 due to the selection of workers.

[7] For the rest of the paper we express all nominal values in dollars using an exchange rate of 6.55 DKK per USD.

3.3.1 Baseline Empirical Specification

Our basic empirical specification relates changes in worker (log) wages and changes in (log) firm TFP controlling for workers and firms characteristics. The primary model we use is then

$$\Delta w_{ijt} = \alpha + \beta S_{jt} + \mathbb{Z}'_{jt}\gamma + X'_{it}\lambda + \delta_t + \varepsilon_{ijt}, \tag{3.1}$$

where Δw_{ijt} is the change of *log* real wages for individual i that works in firm j between periods t and $t-1$, S_{jt} is a measure of firm's productivity changes, $\mathbb{Z}'_{jt}$ is a set of observable firm-level characteristics that might vary over time (industry, lagged firm size, lagged firm age, lagged firm productivity), X'_{it} is a set of individual characteristics (age, education, lagged occupation, lagged experience, lagged tenure, job switch indicator, lagged wage), and δ_t is a year-fixed effect that controls for aggregate economic conditions. The main parameter of interest is β which captures the elasticity of wages to changes in firm-level productivity.[8]

3.3.2 Selection Model

Our primary goal is to separately estimate the effects of TFP changes on workers' wage growth for workers who stay in their firms, and for workers who switch between firms. However, our analysis is likely subject to selection bias, since workers who choose to stay at the firm between periods $t-1$ and t may tend to experience more or less passthrough than those who left would have experienced if they stayed. For example, suppose firms pass negative shocks to workers as productivity decreases. Those workers who receive a large wage drop may choose to leave the firm, which would tend to bias estimates of negative passthrough towards zero if one solely analyzes continuing workers. There may also be selection on the firm side, as firms which experience large negative productivity shocks may choose to exit the market. If these are the firms which tend to have more passthrough, then the estimates of passthrough may be biased. This issue has been mentioned by the previous literature (see for example Guiso et al. (2005)), however, to our knowledge, we are the first to correct for selection on either the worker or firm side in the passthrough literature.

[8]Because our main specification used first-differences in wages and productivity we are implicitly controlling for workers and firms fixed characteristics.

To correct for selection we adopt a simple model to describe job stayers' selection problem as given by

$$
\begin{aligned}
\Delta w_{ijt} =\ & \mathbb{X}_{it}'\Lambda + \varepsilon_{ijt} && \text{if } \ u_{ijt} > 0, \\
\Delta \log w_{ijt} =\ & \text{unobserved} && \text{if } \ u_{ijt} \leq 0, \\
u_{ijt} =\ & Z_{ijt}\delta + \xi_{ijt}, && \\
D_{ijt} =\ & 1 && \text{if } \ u_{ijt} > 0, \\
D_{ijt} =\ & 0 && \text{if } \ u_{ijt} \leq 0.
\end{aligned}
$$

Here u_{ijt} denotes the net utility that a worker gets when she chooses to stay at firm j at time t as opposed to switching to a different firm or out of employment; $w_{ijt}, \mathbb{X}_{ijt}$, are stayers' wage and observable firm/workers characteristics which affect workers' wage growth (same as equation 3.1), and observable characteristics which affect the utility of staying in their job, respectively. When the net utility from staying in their firm is below 0, workers switch out, so we are not be able to observe their within-firm wage change and thus passthrough. We denote whether or not we observe the within firm wage change by the indicator variable D_{ijt}.

Our strategy to correct for the selection problem follows Heckman (1979). Specifically, we assume that the joint distribution for the errors is given by:

$$
\begin{pmatrix} \varepsilon_{ijt} \\ \xi_{ijt} \end{pmatrix} \sim \mathbb{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & \rho\sigma \\ \rho\sigma & 1 \end{pmatrix} \right].
$$

Given this assumption, we estimate a first-stage probit regression of the probability that a given worker stays at her firm as a function of Z_{ijt}, obtaining $\hat{\delta}$. Then we calculate the fitted value of the latent variable $\hat{u}_{ijt}$ and compute the inverse Mills ratio $\hat{\lambda}_{ijt}$ as a function of $\hat{u}_{ijt}$. Finally we include $\hat{\lambda}_{ijt}$ in the second stage regression and get a consistent and unbiased (though not asymptotically efficient) estimator of β.

Our identification strategy then relies on having a reasonable exclusion restriction for the first stage, in that we can include some firm and worker variation which plays a role in the probability that workers will stay or leave their firm, but do not affect the growth rate of workers' wages should they choose to stay at the firm that period. In order to do this, we use the spousal linkage data to create, for each worker, a set of marital status indicators and – for those with working spouses – measures of their spouses employment status. Specifically, we include indicators for marriage status, separation, change of spouse and whether or not the individual's spouse is working if married. This last term is interacted with other spousal information including log wage, change in log wage, firm TFP and log TFP change, age, experience, and whether or not the spouse is a stayer for that period. We exclude spousal working information if the couple is working at the same firm. The assumption for our instrument is that when a worker is getting married/divorced or if his/her spouse has an income change or other employment shock, this will affect the worker's decision on whether or not to keep working at the current firm. However, changes in marriage status, spousal employment,or spousal wages should not affect the worker's wage growth at his/her current firm conditional on staying, unless the couple are working at the same firm. To control for firm selection, we also include in Z_{ijt} various firm-level variables such as financial information and lags of TFP which shouldn't directly affect within-firm worker wage growth conditional on the set of observables in equation 3.1.

3.3.3 TFP Estimation

Most of the literature considers different measures of firm shocks when estimating passthrough, mostly focusing on either raw value added or changes in value-added residuals from an OLS regression of value added on firm characteristics. In contrast, we employ a structural model of firm production and input choice to estimate firm-level total factor productivity (TFP) which we use as our main measure of firm performance. We do this for two reasons. First, we want a measure of firm performance which controls for endogeneity and transmission bias. This is important if we want to separately identify changes in capital stock or worker composition/ability from changes in *firm-level* productivity, while allowing these factors to be potentially correlated. Second, we want to be able to estimate productivity without placing implicit restrictions on the nature of wages or the flexibility of labor. In particular, we want to allow both employment and wages to be responsive to contemporaneous changes in firm productivity. This means we cannot, for example, assume wages perfectly reflect labor quality, or assume perfectly competitive labor markets. Our methodology, which draws on recent work by Gandhi et al. (2018) (hereafter "GNR"), allows for both imperfectly competitive labor markets with adjustment costs and firm-specific wage shocks.

Labor Quality and Wages

In order to identify firm-level productivity in the next section, we want to be able to control for changes in firm-level labor inputs. Our main concern is that if we estimate firm productivity using standard measures of labor inputs (such as a simple count of workers, or number of hours worked), our measure of TFP may include unmeasured differences in workers' quality, driving variation in TFP which may be correlated with wages through this channel rather than the passthrough channel we want to measure. To attempt to control for this and peel out changes in worker quality from our measure of firm productivity, we use worker-side data to construct a quality-adjusted labor input index (the "predicted" wage-bill) to use in our production function estimation.[9]

To construct our quality adjusted-labor input index we proceed as follows. Our firm-side data has information on full-time equivalents working in the firm each year, which we denote E_{jt}. The standard procedure would be to use this directly in the production function estimation, setting $L_{jt} = E_{jt}$. Instead, we use our worker-level information data to construct a firm-level average quality-adjusted labor input index, $\widetilde{G}_{jt}$ which we then multiply by the number of full time equivalents to get our firm-level labor input $L_{jt} \equiv \widetilde{G}_{jt}E_{jt}$. To calculate $\widetilde{G}_{jt}$, we estimate a simple Mincer regression of log hourly wages w_{ijt} on individual characteristics X^m_{ijt} and individual fixed effects a^m_i:

$$w_{ijt} = X^{m\prime}_{ijt}\beta^m + a^m_i + \epsilon^m_{it}. \tag{3.2}$$

We then define a firm's total labor input as $TL_{jt} \equiv \sum_i \widehat{W}_{ijt}H_{ijt}$ where $\widehat{W}_{ijt}$ is the predicted hourly wages (in levels) from the Mincer regression and H_{jt} is the total number of hours

[9] A number of recent papers including Fox and Smeets (2011) argue that controlling for variation in input quality is important for identifying TFP. Fox and Smeets (2011) suggest that using the wage bill as a proxy for worker quality is a possible substitute for controlling for worker quality, especially if individual-level data is not available. However, using the wage bill as the labor input implicitly assumes that wages perfectly represent worker ability, and preclude the ability of firms to adjust wages in response to changes in TFP. To get around this, we use the "predicted" wage-bill based on the fixed effects and observable characteristics of the workers at the firm, which accounts for variation across firms in the wages paid to observably identical workers which may stem from differences in wage contracts, TFP passthrough, imperfect labor markets, etc.

worked for worker i employed by firm j in year t. Using H_{ijt} and information on the average number of hours $\overline{H}$ worked by full time workers in Denmark, we can also construct a worker side measure of a firm's full time equivalents, $\widetilde{E}_{jt} = \sum_i H_{ijt}/\overline{H}$. We do this since we observe hourly wages for most but not all workers in every firm. Note that this implicitly assumes that any workers who are not included in the worker data-side calculation of total quality adjusted labor or FTEs are of the same average quality as the observed workers. The firm's quality-adjusted average labor input is then $\widetilde{G}_{jt} = TL_{jt}/\widetilde{E}_{jt}$. Our measure of labor input L_{jt} then controls for both firm and individual-level changes in ability as measured through the Mincer regression.

Model and Assumptions

We estimate our model on a panel of firms $j \in \mathcal{J}$, where for each firm-year pair we observe output Y_{jt}, capital stock K_{jt}, labor input L_{jt} and intermediate inputs M_{jt}. We will be relying on several timing assumptions to identify the model.

Define $\mathcal{I}_{jt}$ as the information set available to firm j when it enters period t. $\mathcal{I}_{jt}$ contains all of the information relevant to the firm (such as firm productivity) when it makes its period-t decisions. Following GNR, we define any input $X_t \in \mathcal{I}_{jt}$ as *predetermined*. Any such input is thus a function of the previous period's information set: $X_t(\mathcal{I}_{jt-1})$. We will treat capital as a predetermined input. Inputs which are not predetermined (and thus are set in period t) we define as *variable*. We define any input which is variable and where the optimal choice of X_t is a function of lagged values of itself as *dynamic*. We will depart from GNR in assuming that labor is a dynamic input.[10] Finally, an input which is variable but not dynamic we define as *flexible*. Intermediate inputs will be treated as flexible in our framework. This implies that both K_{jt} and $L_{j,t-1}$ are elements of $\mathcal{I}_{jt}$, but L_{jt} and M_{jt} are not.

Here we follow GNR in formally stating the assumptions on the model of firm production.

Assumption 1 *The firm's production function takes the following general form in levels*

$$Y_{jt} = F(K_{jt}, L_{jt}, M_{jt})e^{\nu_{jt}}$$

[10] The recent literature (see Ackerberg et al. (2015)) has argued that allowing labor to be fully flexible introduces significant identification issues. Assuming labor is fully predetermined, as in GNR, would preclude firms of adjusting labor in response to contemporaneous productivity shocks.

and in logs

$$y_{jt} = f(k_{jt}, \ell_{jt}, m_{jt}) + \nu_{jt}$$

where f is a crs and differentiable function which is strictly concave in m_{jt}.

Assumption 2 *Capital ($K_{jt} \in \mathcal{I}_{jt}$) is predetermined and a state variable. Labor input ($L_{jt} \notin \mathcal{I}_{jt}$) is dynamic, such that $L_{jt-1} \in \mathcal{I}_{jt}$ is a state variable. Intermediate inputs ($M_{jt} \notin \mathcal{I}_{jt}$) are flexible, so that $M_{jt-1} \notin \mathcal{I}_{jt}$.*

The Hicks-neutral productivity term ν_{jt} can be decomposed into a persistent component ω_{jt} which is known to the firm when it makes input decisions, and a transitory component ε_{jt} which is unknown to the firm when making input decisions.

Assumption 3 *The permanent productivity term $\omega_{jt} \in \mathcal{I}_{jt}$ is observed by the firm prior to making period-t decisions and is first-order Markov, such that $\mathbb{E}[\omega_{jt}|\mathcal{I}_{jt-1}] = \mathbb{E}[\omega_{jt}|\omega_{jt-1}] = h(\omega_{jt-1})$ for some continuous function $h(.)$. $\varepsilon_{jt} \notin \mathcal{I}_{jt}$ is i.i.d across firms and time, with $P_\varepsilon(\varepsilon_{jt}|\mathcal{I}_{jt}) = P_\varepsilon(\varepsilon_{jt})$.*

We normalize $\mathbb{E}[\varepsilon_{jt}] = 0$ and define $\eta_{jt} = \omega_{jt} - \mathbb{E}[\omega_{jt}|\omega_{jt-1}]$ which implies $\mathbb{E}[\eta_{jt}|\mathcal{I}_{jt-1}] = 0$. This gives us several measures of change in total firm productivity $\nu_{jt} = h(\omega_{jt-1}) + \eta_{jt} + \varepsilon_{jt}$. $h(\omega_{jt-1}) - \omega_{jt-1}$ is the *expected and persistent* change in productivity, η_{jt} is the *unexpected and persistent* shock to productivity, and $\varepsilon_{jt} - \varepsilon_{jt-1}$ is the *unexpected and transitory* change in total productivity.

Assumption 4 *We assume that demand for intermediate input $m_{jt} = \mathcal{M}(k_{jt}, \ell_{jt}, \omega_{jt})$ is strictly monotone in ω_{jt}.*

Note that this conditional (on period-t labor) demand function is critical in identifying the production function while allowing labor to be a dynamic (and not predetermined) input. It allows allows for labor adjustment costs and firm-specific wage shocks, both of which may be important in our setting.[11] We also make the following assumption about firm's profit maximizing behavior and environment:

[11] See Ackerberg et al. (2015) and GNR.

Assumption 5 *Firms maximize short-run expected profits and are price takers in both output and intermediate input markets. Denote the common output price index for period t as P_t and the common intermediate price index as ρ_t.*

This framework gives us all of the tools to obtain robust estimates of TFP which satisfy our two main goals.

Identification and Estimation

Following GNR, assumptions 1 to 5 give us the following first order condition for firm's profit maximization problem in period t with respect to M_{jt}:

$$P_t \frac{\partial}{\partial M_{jt}} F(K_{jt}, L_{jt}, M_{jt}) e^{\omega_{jt}} \mathcal{E} = \rho_t$$

where $\mathcal{E} \equiv \mathbb{E}[e^{\varepsilon_{jt}}]$ is a constant. Multiplying both sides by M_{jt}/Y_{jt}, plugging in the production function and rearranging provides our first estimating equation:

$$\begin{aligned} s_{jt} &= \ln \mathcal{E} + \ln D(k_{jt}, \ell_{jt}, m_{jt}) - \varepsilon_{jt} \\ &\equiv \ln \left(D^{\mathcal{E}}(k_{jt}, \ell_{jt}, m_{jt}) \right) - \varepsilon_{jt} \end{aligned} \tag{3.3}$$

where $s_{jt} \equiv \ln(\rho_t M_{jt}/P_t Y_{jt})$ is the log revenue share of intermediate input expenditure and $D(k_{jt}, \ell_{jt}, m_{jt}) \equiv \frac{\partial}{\partial m_{jt}} f(k_{jt}, \ell_{jt}, m_{jt})$ is the output elasticity of materials. Since by assumption 3 we have $\mathbb{E}[\varepsilon_{jt}] = 0$, we can use equation 3.3 to identify ε_{jt} and $D^{\mathcal{E}}$.
Given $\varepsilon_{jt} = \ln \left(D^{\mathcal{E}}(k_{jt}, \ell_{jt}, m_{jt}) \right) - s_{jt}$, we can identify the constant $\mathcal{E} = \mathbb{E}[\exp(\varepsilon_{jt})]$, which subsequently provides the elasticity $D(k_{jt}, \ell_{jt}, m_{jt}) = D^{\mathcal{E}}(k_{jt}, \ell_{jt}, m_{jt})/\mathcal{E}$. Once we know $D(k_{jt}, \ell_{jt}, m_{jt})$ and ε_{jt}, we can estimate the rest of the production function non-parametrically. Then we have

$$\mathcal{D}(k_{jt}, \ell_{jt}, m_{jt}) \equiv \int \frac{\partial}{\partial m_{jt}} f(k_{jt}, \ell_{jt}, m_{jt}) dm_{jt} = f(k_{jt}, \ell_{jt}, m_{jt}) + \mathit{\Psi}(k_{jt}, \ell_{jt}) \tag{3.4}$$

Define $\tilde{y}_{jt} \equiv y_{jt} - \varepsilon_{jt} - \mathcal{D}(k_{jt}, \ell_{jt}, m_{jt}) = -\mathit{\Psi}(k_{jt}, \ell_{jt}) + \omega_{jt}$. Plugging in the structure of ω_{jt} from assumption 3, we get our second estimating equation:

$$\tilde{y}_{jt} = -\mathit{\Psi}(k_{jt}, \ell_{jt}) + h(\tilde{y}_{jt-1} + \mathit{\Psi}(k_{jt-1}, \ell_{jt-1})) + \eta_{jt} \tag{3.5}$$

where $\tilde{y}_{jt}$ is observable given the stage-one estimates of ε_{jt} and $\mathcal{D}(k_{jt}, \ell_{jt}, m_{jt})$. Our assumptions on the firm's information set give us $\mathbb{E}[\eta_{jt}|k_{jt}, \ell_{jt-1}, k_{jt-1}, \tilde{y}_{jt-1}, \ell_{jt-2}] = 0$, which we use with equation 3.5 to identify Ψ, h, and thus η_{jt}.[12]

Our estimation procedure follows GNR in using a standard sieve-series estimator to non-parametrically identify the input elasticity and production function. We proceed in two steps. First, we estimate the share equation with a complete 2nd degree polynomial in k_{jt}, ℓ_{jt} and m_{jt} using nonlinear least squares. This estimator solves

$$\min_{\gamma'} \sum_{j,t} \varepsilon_{jt}^2 = \sum_{j,t} \left[s_{jt} - \ln \left(\sum_{r_k + r_\ell + r_m \leq 2} \gamma'_{r_k, r_\ell, r_m} k_{jt}^{r_k} \ell_{jt}^{r_\ell} m_{jt}^{r_m} \right) \right]^2 \tag{3.6}$$

which gives us estimates of $\hat{\varepsilon_{jt}}$ and $\widehat{D^{\mathcal{E}}}(k_{jt}, \ell_{jt}, m_{jt}) = \sum_{r_k + r_\ell + r_m \leq 2} (\hat{\gamma}'_{r_k, r_\ell, r_m} k_{jt}^{r_k} \ell_{jt}^{r_\ell} m_{jt}^{r_m})$. We can then recover $\widehat{\mathcal{E}} = \mathbb{E}[\exp(\hat{\varepsilon}_{jt})]$ and the input elasticity

$$\widehat{D}(k_{jt}, \ell_{jt}, m_{jt}) = \sum_{r_k + r_\ell + r_m \leq 2} (\hat{\gamma}_{r_k, r_\ell, r_m} k_{jt}^{r_k} \ell_{jt}^{r_\ell} m_{jt}^{r_m})$$

where $\hat{\gamma} \equiv \hat{\gamma}' / \widehat{\mathcal{E}}$. We then integrate the estimated flexible input elasticity to recover

$$\widehat{\mathcal{D}}(k_{jt}, \ell_{jt}, m_{jt}) = \sum_{r_k + r_\ell + r_m \leq 2} \left(\frac{m_{jt}}{r_m + 1} \hat{\gamma}_{r_k, r_\ell, r_m} k_{jt}^{r_k} \ell_{jt}^{r_\ell} m_{jt}^{r_m} \right)$$

which allows us to recover $\hat{\tilde{y}}_{jt} = y_{jt} - \hat{\varepsilon}_{jt} - \widehat{\mathcal{D}}(k_{jt}, \ell_{jt}, m_{jt})$. In the second step, we estimate equation 3.5 using GMM, where we similarly approximate $\Psi(k_{jt}, \ell_{jt})$ using a complete 2nd degree polynomial and $h(\omega_{jt-1})$ as a 1st degree (linear) polynomial, implying that persistent TFP follows an AR(1) process. Since we can identify both the constant of integration and TFP only up to an additive constant, we follow GNR in normalizing Ψ to be mean zero and so allow the constant to show up in the level of productivity. This gives us the following

[12] This differs from GNR, who assume that labor is predetermined. We relax this assumption since we want to allow firms to adjust labor in response to persistent shocks in productivity (η_{jt}).

second-stage estimating equation:

$$\tilde{y}_{jt} = -\sum_{0<\tau_k+\tau_\ell\leq 2} \alpha_{\tau_k,\tau_\ell} k_{jt}^{\tau_k} \ell_{tj}^{\tau_\ell} + \sum_{0\leq a\leq 1} \delta_a \left(\tilde{y}_{jt-1} + \sum_{0<\tau_k+\tau_\ell\leq 2} \alpha_{\tau_k,\tau_\ell} k_{jt-1}^{\tau_k} \ell_{tj-1}^{\tau_\ell} \right)^a + \eta_{jt}. \quad (3.7)$$

Since $\mathbb{E}[\eta_{jt}|k_{jt}, \ell_{jt-1}, \mathcal{I}_{jt-1}] = 0$, the only endogenous variable is ℓ_{jt}. Thus we can use functions of the set $\{k_{jt}, k_{jt-1}, \ell_{jt-1}, \ell_{jt-2}, m_{jt-1}, \tilde{y}_{jt-1}\}$ as instruments. In particular, our moments are $\mathbb{E}[\eta_{jt}\tilde{y}_{jt-1}^a]$ and $\mathbb{E}[\eta_{jt}k_{jt}^{\tau_k}\ell_{jt-2}^{\tau_\ell}]$ for all $0 \leq a \leq 1$ and $0 < \tau_k + \tau_\ell \leq 2$, leaving us exactly identified.[13] This provides us with estimates of the production function parameters as well as $\hat{\omega}_{jt}$, $\hat{\eta}_{jt}$ and $\hat{\hat{\omega}}_{jt} \equiv \hat{h}(\hat{\omega}_{jt-1}) = \hat{\delta}_0 + \hat{\delta}_1 \hat{\omega}_{jt-1}$.

3.4 Empirical Results

3.4.1 Non-Parametric Analysis

We start our analysis by discussing the relation between changes in firm-level productivity and workers' wage growth using a simple, non parametric approach. For doing that, we pool our sample of firms and individuals between 1995 and 2012 and we sort firms by their productivity growth in one hundred bins. Then, within each bin, we calculate different moments of the wage growth distribution: the mean, the standard deviation, and the 90th, 50th, and 10th percentiles. Moments are weighted by firm's employment so as to reflect the underlying firm size distribution within each bin.

The left panel of figure 3.3 shows the average wage growth within each bin of the TFP growth distribution. Three aspects of the plot are worth noticing. First, the average wage growth is remarkably stable from the first percentiles of the distribution - where the typical firm experiences a large decline in TFP - up to the 80th percentile of the distribution. This suggests that most firms provide insurance to their workers from changes in firm-level productivity. Second, the average wage growth increases dramatically as we move above the fifth quintile of the TFP growth distribution. The typical firm above the 80th percentile of the TFP growth distribution experiences an increase of TFP of 25% which translates into an average wage growth of 20% for the workers of these firms. Finally, considering

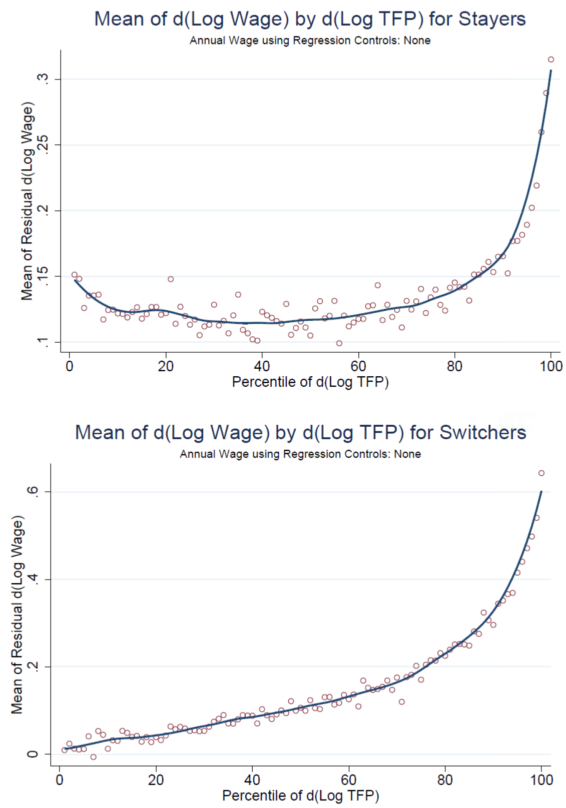
[13] As pointed out by GNR, this implies that the estimator is a sieve-M estimator, which allows us to do inference treating the polynomials as if they were the true parametric structure.

how flat the relation between wage growth and TFP growth is across most of the TFP growth distribution, it is not surprising that most papers in the literature find very small average passthrough from TFP to wages.

How does wage growth vary for workers that switch between firms? To provide a first answer to this question, the right panel of figure 3.3 repeats the previous analysis by calculating the average wage growth for switchers within each percentile of the TFP growth distribution. Importantly, the TFP growth percentiles in the x-axis in both plots of figure 3.3 are the same so we can directly compare them. Still, the interpretation of a TFP change is slightly different. In the left panel, a TFP change reflects the change in the productivity of the same firm across time whereas in the right panel, a TFP change reflects the gap in productivity between two different firms across time. We discuss and control for this in the analysis below. The right panel of figure 3.3 shows two important results, first, workers that move between firms whose TFP differential implies a growth rate at the left tail of the TFP growth distribution obtain almost no wage growth. Moreover, these workers observe lower wage growth than the average worker that stays in the same firm conditional on the firm experiencing the same decline in TFP. Second, workers that move to more productive firms do experience a wage growth, which is higher than the wage growth of workers that stay in the same firm conditional on the firm observing the same TFP growth. In fact, the average wage growth for stayers and switchers crosses at around the 40th percentile of the TFP growth distribution.

We also find significant differences in the dispersion of wage growth across the TFP growth distribution. In fact, firms at the top and bottom of the distribution of TFP growth command higher wage growth dispersion for their workers relative to firms in the middle of distribution. To see this, the left panel (right panel) of figure 3.4 shows the 90th-to-10th percentiles spread of the wage growth distribution within different percentiles of the TFP growth distribution for workers that stay in the same firm (switch across firms). The figure shows a marked u-shaped pattern with more dispersion at the top and bottom. In fact, dispersion of wage growth at lower percentiles of the TFP growth distribution is almost 30 percentage points higher than the dispersion of wage growth among workers in firms at the middle of the TFP growth distribution. The difference is even more stark between workers in the middle and at the top of the TFP growth distribution: relative to the middle of

Figure 3.3: Average Wage Growth Across the TFP Growth Distribution

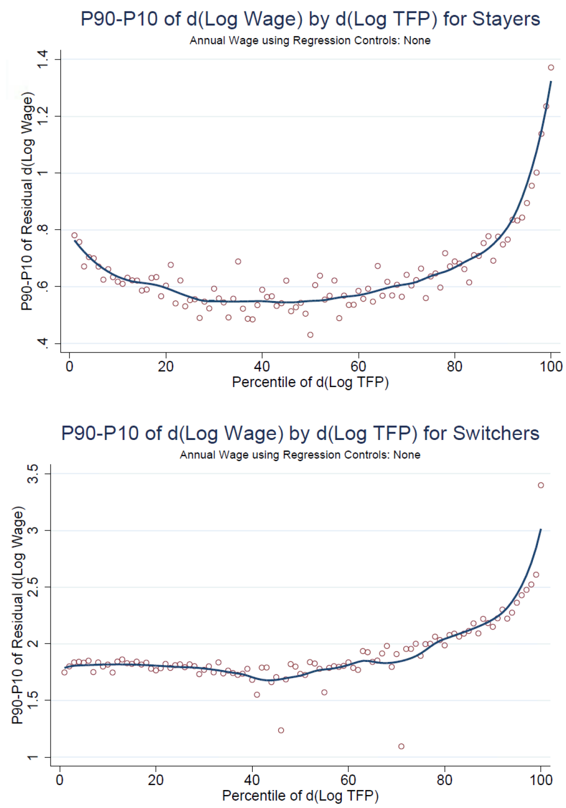


Note: The left panel of figure 3.3 shows the employment weighted average of the workers' wage growth distribution within each percentile of the TFP growth distribution for a sample of workers that stay in the same firm for two consecutive period. The right panel displays the same statistic for the set of worker that switch firms between two consecutive periods.

the distribution, dispersion of wage growth almost doubles at the highest percentiles of the TFP growth distribution.

In summary, our simple non parametric analysis shows a substantial heterogeneity of firm insurance across firms and positive relation between productivity growth and wage growth. Still, this heterogeneity can be the product of differences across workers, across firms, and over time, which cannot be easily captured by the simple setting we have discussed. Moreover, selection of workers and firms into different groups might impact our results. In the following section we control for observable characteristics of stayers and switchers to

Figure 3.4: Dispersion of Wage Growth Across the TFP Growth Distribution



Note: The left panel of figure 3.4 shows the employment weighted 90th-to-10th percentiles spread of the workers' wage growth distribution within each percentile of the TFP growth distribution for a sample of workers that stay in the same firm for two consecutive period. The right panel displays the same statistic for the set of worker that switch firms between two consecutive periods.

show that passthrough is positive and economically significant.

3.4.2 Passthrough from Productivity Shocks to Wages

Baseline Results

In this subsection we analyze the passthrough of firms' shocks to workers' wage growth. We focus on a simple regression analysis for two reasons. First, this simple approach is similar to what has been used in the previous literature, allowing us to more directly compare our results with other papers. Second, we aim to highlight that even if one puts aside selection considerations, the passthrough from TFP shocks to workers' wages is positive and economically significant.

As described in section 3.3, our baseline analysis is based in a series of OLS panel regressions of the form,

$$\Delta \log w_{ijt} = \alpha + \beta^{\eta}\eta_{ijt} + \beta^{\varepsilon}\varepsilon_{ijt} + \mathbb{Z}'_{jt}\gamma + X'_{it}\lambda + \delta_t + \epsilon_{ijt} \tag{3.8}$$

where the main coefficients of interest are β^{η} and β^{ε}, the elasticity of wages to changes in persistent and transitory shocks to firm-level productivity. In this section we restrict our sample to stayers, who are workers employed at the same firm in periods $t-1$ and t.

Table 3.2 displays our first set of results. Column (1) shows that the passthrough from TFP shocks to wage growth is positive and statistically significant for stayers. This is true for both the persistent component of the TFP shock as well as the transitory component. Our baseline results indicate that a 10% increase in the persistent component of firm's TFP drives an increase of 0.16% in workers wages. The same change in the transitory component generates an increase in wages of 0.4%. Hence, our results suggest that both types of productivity shocks, transitory and persistent, have a significant impact on workers wages.

We evaluate the monetary value of firm shocks by simply multiplying the change in the average wage generated by a positive productivity shock of one standard deviation. The column labeled Value in table 3.2 shows that a shock of one standard deviation in the persistent component of firm TFP implies a change in annual income of around \$251 USD for a worker making the average wage. A one standard deviation transitory innovation in firm-level TFP implies a change in annual income of around US\$792. Together they represent 2% of the overall income per capita in Denmark. Considering that in any given year around 33% of firms (which employ around 40% of all the workers in the economy) receive either a persistent or transitory productivity shock that is at least one standard deviation away from the mean (6% of firms and 8% of workers experience both), this represents a significant aggregate change in income. Moreover, our measure of wages reflects only changes in the individual's wage rate. Using a more standard measure of income, one that includes changes in the number of hours, would increase the elasticity of workers' wages to firms' shocks.

Does the sign of the change in TFP matter for the passthrough from TFP to wages?

To answer this question we separate workers employed by firms experiencing a negative productivity shocks from those workers in firms experiencing a positive productivity shocks. We then run the same specification in equation (3.8) within each group of workers. Columns (2) and (3) of table 3.2 show the results. First, notice in column (2) that the coefficient on negative permanent TFP shock changes to basically zero and is not statistically significant, suggesting that firms insulate workers from negative shocks to persistent productivity. Column (3) however, shows a significant and positive correlation between positive shocks to persistent TFP and wage growth, indicating that firms pass a fraction of positive changes in productivity to wages. In monetary values, a change in permanent TFP shock of one standard deviation, conditional on this change being negative, translates into a decline in annual labor earnings of only \$16 for a worker with the average wage in that group, whereas a positive change in TFP translates into an increase in annual labor earnings of \$671 for that same worker. The effect of transitory TFP shocks on wages is significant for both negative and positive shocks, showing a similar asymmetric effect to the persistent shocks.

The positive but small relationship between negative and persistent shocks to firm's TFP and wage growth that we find falls in line with results found in other papers (see for instance Juhn et al. (2018) and Rute Cardoso and Portela (2009)). These results are based on the sample of workers that stay in the same firm for two consecutive periods. However, these regression results might be biased by the presence of selection into the status of "stayer". We discuss this in the next section and implement a simple method to correct for the effect of selection on the passthrough coefficients.

Selection

As we discussed in section 3.3.2 selection might have a substantial impact in measuring the effects of firms' shocks to workers' wages. This problem arises because the sample considered in the first three columns of table 3.2 is defined only for continuing workers who have stayed at same firm, neglecting the possibility that the probability of staying may be correlated with passthrough and expected change in wages.[14] For instance, after

[14] In Appendix C.1, we investigate the effect of firm productivity shocks on the probability of worker entry and exit, confirming that firm-level TFP shocks do drive worker flows in and out of the firm. The direction of the bias, however, is not ex-ante clear.

Table 3.2: Passthrough is Positive and Significant for Transitory and Persistent Shocks

Dependent Variable:	Log change of real annualized wages, $\Delta w_{i,j,t}$								
Sample:	Stayers						Switchers		
Selection:	Baseline			Corrected			Corrected		
	(1) ΔTFP_{jt}	(2) $-\Delta TFP_{jt}$	(3) $+\Delta TFP_{jt}$	(4) ΔTFP_{jt}	(5) $-\Delta TFP_{jt}$	(6) $+\Delta TFP_{jt}$	(7) ΔTFP_{jt}	(8) $-\Delta TFP_{jt}$	(9) $+\Delta TFP_{jt}$
Persistent (β^{η})	0.016***	0.001	0.044***	0.045***	0.035***	0.088***	0.024***	0.034***	0.015***
	(0.004)	(0.004)	(0.009)	(0.005)	(0.006)	(0.008)	(0.005)	(0.007)	(0.003)
Transitory (β^{ε})	0.041***	0.035***	0.056***	0.042***	0.033***	0.057***	0.001	0.004*	−0.005**
	(0.005)	(0.003)	(0.013)	(0.005)	(0.003)	(0.013)	(0.002)	(0.003)	(0.002)
N	4,066,293	2,904,326	1,161,967	4,066,293	2,904,326	1,161,967	237,911	124,703	113,208
R^2	0.118	0.118	0.122	0.118	0.118	0.122	0.326	0.349	0.300
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Fixed Effect	Year	Year	Year	Year	Year	Year	Year	Year	Year
$Value ($\beta^{\eta}$)	$251	$16	$671	$705	$537	$1,326	$5,253	$9,142	$3,299
$Value ($\beta^{\varepsilon}$)	$792	$690	$1053	$812	$624	$1,059			
% Income (β^{η})	0.5%	0.0%	1.3%	1.4%	1.1%	2.6%	10.0%	13.9%	6.1%
% Income (β^{ε})	1.5%	1.3%	2.1%	1.6%	1.2%	2.1%			

Note: Table 3.2 shows a set of OLS panel regressions controlling for firm and worker characteristics. In columns (1) to (6) the dependent variable is the growth rate of real wages for individuals that stay in the same firm for two consecutive periods (stayers). In columns (7) to (9) the dependent variable is the growth rate of real wages for individuals that switch between firms between firms in consecutive periods (switchers). The main explanatory variables for stayers are measures of the change in the persistent shock to TFP and the transitory shock to firm-level productivity; For switchers are the difference in the persistent and transitory shocks of TFP between the old and new firms. The row named Value (USD) shows the change in real wages resulting from a one standard deviation shock to the corresponding shock in TFP for a worker with the average wage for that group. $*p < 0.1, **p < 0.05, ***p < 0.01$. Robust standard errors are clustered at the firm level.

a negative productivity shock firms might decide not to reduce wages but lay off some workers to reduce labor costs. Since these workers are not counted as stayers, we do not consider the effect of changes in TFP on their wages, effectively reducing the measured impact of negative changes of productivity on wages. Workers who are most exposed to or expect large passthrough from negative shocks may also voluntarily choose to leave the firm. Similarly, the effect of positive productivity shocks might be downward biased. Workers facing large passthrough effects from a positive TFP shock may be very high skilled workers who are more likely to be poached by, or leave for, other firms. In this section we make some progress in controlling for selection using the simple Heckman selection method outlined in section 3.3.2.

Columns (4) to (6) of table 3.2 show that the results after correcting for selection are quite different. First of all, the coefficient for η_{ijt} in column (4) almost triples in magnitude relative to the impact of persistent TFP shocks on wages when one does not control for selection (compare to column (1)). The selection-corrected results imply that a worker in a firm that experiences a persistent shock to TFP of one standard deviation will receive an average increase in her annual earnings of \$705, or 1.4% of the average annual income. This increase in the passthrough from firms' productivity to workers' wages points to a much smaller role for the firm as a source of insurance for the worker. This holds true for both positive and negative shocks. Starting from column (5), the effect of a negative productivity change goes from being near zero in column (2) to 0.035 in column (6) and it becomes statistically significant. This implies that a worker in a firm receiving a negative permanent productivity shock of one standard deviation sees his annual income reduced by \$537, a value that is much larger relative to the value measured when one does not control for selection. We also find a significant increase in the effect of a positive permanent productivity shock into wages as it is shown in column (6) of table 3.2. In this case, a positive persistent productivity shock of one standard deviation commands an increase in wages of more than \$1,000, that is, almost 2.6% of the average income per capita.

There are several conclusions we can draw from these results. First, there is asymmetry in worker exposure to firm-level TFP shocks. Firms appear to pass positive shocks to workers more than they pass negative shocks, providing some insurance against movements in wages. Second, it seems that selection biases the estimated passthrough coefficients downwards on average, as both the overall and negative shock passthrough coefficients increased dramatically after correcting for selection. This confirms our intuition that both workers and firms may exit when faced with big negative TFP shocks or threats of significant negative passthrough. However, there is also a positive selection bias, with the effect of a positive shock doubling after correction. This is possibly due to better workers at firms experiencing TFP growth leaving for better opportunities. The transitory shock component, however, is not affected when correcting for the selection bias. This is intuitive and fits with the underlying model of firm optimization – firms and workers make their employment decisions in period t with information on the persistent shock to TFP η_{jt}, but not the transitory shock ε_{jt}. Workers can not ex-ante predict and react to the transitory component of TFP, so selection has almost no effect on the transitory shock coefficient.

To sum up, firms TFP shocks have a sizable and significant passthrough effect on worker wage growth. After carefully correcting for selection bias, the joint effects of TFP shocks on wage growth is much bigger than commonly found in the literature. To put into context, a one standard deviation shock to both the permanent and transitory component is associated with a more than $1,500 change annual income for a worker with the average wage, or 2% of per capita income. Importantly, passthrough is not symmetric: firms pass more of their positive persistent and transitory shocks on to workers than their negative shocks. However, negative passthrough is significant and not negligible – firms are not providing full insurance or near full insurance to workers when they encounter a negative TFP shock.

Switchers and Stayers

So far we have focused on the effect of TFP shocks on stayers, that is, workers that maintain a stable employment relationship with a firm for the two years over which the change in TFP is calculated. This is a natural starting point as changes in wages for continuing workers can be tied more easily to changes in firm productivity and concepts of insurance against firm-level risk. Moreover, this is the group of workers that the literature has analyzed more often, ignoring the effect of firm shocks on the wages of workers that move between firms. In this section we extend the existing literature to take into account the effect of idiosyncratic, firm and worker-level, productivity changes on the wages of those workers that move across different employers. This is a large group of workers: in any given year around 20% of Danish workers changed employer. Unfortunately, our data does not allow us to directly distinguish between an individual who passed through an unemployment spell prior to joining a different employer or had a direct transition between employers. Therefore, we will put aside issues related to voluntary or involuntary separations and we will treat all workers who make annual employment-to-employment transitions the same.

Similar to the previous section, we run a set of OLS panel regressions in which the dependent variable is the change in real wages for a individual between two consecutive years and the independent variable is the change in the TFP of the firm in which the individual works. Notice that for switchers the interpretation of a positive or negative productivity shock is different than for stayers. For the latter group, it represents a productivity change for the firm in which they work, whereas for switchers it also captures the difference in productivity between two different firms. Hence, a positive TFP change for a switcher indicates that the individual moved to a firm with higher TFP relative to the firm at which she used to work, and this change is independent of the actual change in productivity experienced by any of the firms. For instance, it is possible that the transition was motivated by a productivity decline in the firm of origin, or an increase in the productivity of the new firm that poached the worker, or both. To capture these effects we include in the regression the shocks to the productivity of both of the firms the individual is transitioning[15] In particular, we modify the model in equation (3.1) as follows,

$$\Delta w_{ijkt} = \alpha + \beta_1^{\eta}\eta_{jkt} + \beta_2^{\eta}\eta_{kkt} + \mathbb{Z}'_{jt}\gamma + X'_{it}\lambda + \delta_t + \varepsilon_{ijkt}, \tag{3.9}$$

where Δw_{ijkt} is the change in real log hourly wages of an individual that works in firm j and moved from firm k. η_{jkt} is defined as the unexpected in period $t-1$ innovation to the firm TFP at which the worker is employed. Specifically, $\eta_{jkt} \equiv \omega_{jt} - \mathbb{E}[\omega_{kt}|\omega_{kt-1}]$. η_{kkt} is the unexpected in $t-1$ innovation to persistent productivity in the firm which the worker left. As before, the main coefficient of interest is β_1^{η} which reflects the elasticity of a change in wages as a response to a shock in persistent TFP for the individual.

The right panels of table 3.2 show the results. Columns (7) to (9) show that the effect of TFP changes on wages of switchers is much stronger than it is for stayers. Furthermore, the

[15] Notice that this distinction is irrelevant for stayers as the firm of origin and destination is the same firm.

large difference in dollar values that is associated with the shock to persistent TFP is largely due to the differences in the standard deviation of TFP changes for stayers and switchers, as well as their difference in average wages. For example, the elasticity of wage growth to persistent TFP shocks is almost the same between stayers and switchers when they face negative shocks (0.035 vs 0.034), but the average wage loss from a one standard deviation negative TFP shock is \$537 dollars for stayers vs \$9,142 dollars for switchers, or 1.1% of annual average income vs 13.9% annual average income. This stark difference between stayers and switchers is because that standard deviation of within-firm TFP changes for stayers is about 0.4, but about 4 for switchers – job switchers experience 10 times more variance in persistent TFP between years than job stayers. This makes sense since stayer TFP changes reflect the same firm's TFP growth, while TFP changes for switchers reflect the differences between the old and new firm, which can be significant. Furthermore, the average annual wage for stayers who see a negative TFP shock is \$48,800 dollars, while switchers the average is \$65,800 dollars. Therefore the same passthrough parameter is associated with drastically different wage losses for stayers and switchers.

3.5 Heterogeneous Passthrough

In this section we study how the passthrough from changes in firm TFP to worker wage growth varies across the distribution of worker and firm types. We focus on how passthrough differs across worker age and income levels, across firm productivity levels, and across the business cycle. We also investigate how wages are affected for workers moving between firms of differing productivity.

3.5.1 Worker Side Heterogeneity

High versus Low Wage Workers

We first study how passthrough from firm shocks to worker wages varies across workers of different income levels. We split our sample of stayers into two groups. First, we classify workers in a given year as "low income" if their labor earnings are below the 20th percentile of the labor earnings distribution in that year and we classify workers as "high income" if their labor earnings are above the 80th percentile for that year. We then estimate the effect of shocks to firm TFP on wages within each of these groups, correcting for selection as described in section 3.4.2.

The results both with and without selection correction are shown in table 3.3. The differences between low and high wage workers are staggering: selection issues aside, the response of wages to changes in firm-level TFP are, on average, much higher for high wage workers relative to low wage workers. For instance, comparing columns (1) and (4) in the top panel of table 3.3 (panel A) we see that a shock in the persistent component of TFP of one standard deviation implies a change in worker earnings of about $2,000 more if the individual is a high wage worker relative to a low wage worker. Even after considering that high wage workers receive labor earnings that are in average two-times higher than the income of low wage workers, the effect on high wage workers is still much bigger. In fact, a change in income associated with one standard deviation of permanent TFP shock is about 3.3% in the annual earnings for high wage workers compare to 2.0% annual earnings for low wage workers.[16] We also find large differences when comparing the effects of positive and negative shocks. Looking at columns (2) and (3) for instance, we see that the elasticity of negative shocks is roughly two thirds the wage elasticity for positive shocks among high wage workers. We observe similar results for low wage workers.[17]

Note that for both high wage worker and low wage workers, the passthrough effect from persistent TFP shocks to wage growth is much higher than it is for average wage workers (column 4 to 6 in table 3.2). This is suggests that middle wage workers (those whose income ranges from the 20 to 80 percentiles) are the least sensitive to firm TFP shocks. Finally, the level of transitory shock passthrough is similar for high and low wage workers.

Young and Old Workers

Workers may be more or less exposed to firm TFP shocks depending on their age. One might expect that older workers are likely to be more experienced on average or have greater tenure and therefore are potentially more insured by firms than workers who have just entered the firm. Differences in passthrough across age may also be due to age-related

[16] Real average annual income for high wage workers is around $85,000 whereas for low wage workers is around $35,000.

[17] As before, selection has an important impact on the estimates more so for negative shocks and for high wage workers – who also appear to be significantly more exposed to income risk due to negative firm shocks than low wage workers.

Table 3.3: Passthrough is Highly Heterogeneous Across Worker Types

A	(1)	(2)	(3)	(4)	(5)	(6)
	High Wage Workers			**Low Wage Workers**		
	ΔTFP_{jt}	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$	ΔTFP_{jt}	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$
Persistent (β^{η})	0.109***	0.110***	0.154***	0.067***	0.047***	0.096***
Transitory (β^{ε})	0.058***	0.044***	0.080***	0.045***	0.042***	0.047***
\$Value (β^{η})	\$2,669	\$2,702	\$3,759	\$621	\$436	\$873
% of Income (β^{η})	3.3%	3.3%	4.6%	2.0%	1.4%	2.9%

B	**Young Workers**			**Old Workers**		
	ΔTFP_{jt}	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$	ΔTFP_{jt}	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$
Persistent (β^{η})	0.116***	0.114***	0.205***	0.041***	0.031***	0.074***
Transitory (β^{ε})	0.050***	0.040***	0.060***	0.039***	0.028***	0.059***
\$Value (β^{η})	\$1,594	\$1,590	\$2,702	\$654	\$520	\$1,158
% of Income (β^{η})	3.5%	3.4%	6.2%	1.2%	1.0%	2.2%

Note: Table 3.3 shows a set of panel regressions controlling for firm and worker characteristics. In each column, the dependent variable is the growth rate of real wages for individuals that stay in the same firm for two consecutive periods. The main explanatory variables are estimated shocks to the transitory and persistent components of firm-level (log) TFP. The row named Value (USD) shows the change in real wages resulting from a one standard deviation shock to TFP for a worker with the average wage for that group. The top panel shows estimates without correcting for selection while the bottom panel shows selection-corrected estimates. High wage: top 20% of wage distribution; Low wage: bottom 20% of wage distribution. $*p < 0.1, **p < 0.05, ***p < 0.01$. Robust standard errors are clustered at the firm level.

selection into high or low passthrough firms, industries or occupations. We divide our sample of workers by age, and define workers who are younger than 35 years old as “young workers”, and define workers who are older than 50 as “old workers”. The results are shown in the bottom panel of table 3.3. As expected, the effect of persistent TFP shocks on wage growth is stronger for younger workers than older workers. The passthrough elasticity for younger workers is almost three times that for older workers. In dollars, the average younger worker experiences a more than \$1,500 dollar wage change from a one standard deviation shock to persistent TFP, while for older workers, this number is \$654. This is an large effect for younger workers considering that their average annual wage is almost \$10,000 dollars lower than for older workers. Similar with previous results, workers are relatively more sheltered from negative shocks than from positive shocks, but negative passthrough is still significant. As we found with high and low wage workers, there is not nearly as much heterogeneity in transitory shock passthrough as there is for persistent shock passthrough, though passthrough from transitory shocks are still economically and statistically significant.

Long Term Effects of Shocks

So far we have focused on the effect of transitory or persistent productivity shocks on one-year wage change. Hence, a natural question is whether the passthrough we have documented so far in our analysis represents transitory or permanent shocks to worker wages. This is important as firm productivity shocks that translate to permanent changes in worker wages represent a source of risk that is more difficult to insure against. To look at this we run regressions of the form

$$\Delta \log w_{ijt} = \alpha + \beta^{\eta} \eta_{j,t} + \mathbb{Z}'_{jt}\gamma + X'_{it}\lambda + \delta_t + \varepsilon_{ijt}, \tag{3.10}$$

where, unlike our basic specification in equation 3.1, the dependent variable is defined as $\Delta \log w_{ijt} = \log w_{ijt+3} - \log w_{ijt-1}$. Hence, the coefficient β^{η} captures the long lasting effect on wages of a change in the persistent shock to firm's productivity, η. Thus we are looking at the effect of a shock in TFP in period t on the total change in wages from $t-1$ to $t+3$. Table 3.4 shows the results for several dimensions of heterogeneity. First it is clear that productivity shocks at the firm level have a long lasting impact on workers wages and this effect greatly differs across groups. For instance, a high wage worker of a firm that experiences a positive change in productivity of one standard deviation between periods $t-1$ and t will have gained by period $t+3$ a total of \$5,800 USD more than an similar worker at an otherwise identical firm with no TFP shock. The effect for low wage workers is smaller but still significant as a change of one standard deviation in productivity generates a 1% increase in their wages after a year. Negative productivity shocks at the firm level also have long lasting effect on workers wages, specially for high wage workers and young workers.[18]

[18]Our results also suggest a significant role for transitory shocks – the $\epsilon_{j,t}$ in the firm's productivity

Table 3.4: Productivity Shocks Have Long Lasting Effects on Worker Wages

	(1)	(2)	(3)	(4)	(5)	(6)
A		**Stayers**			**Expansion**	
	ΔTFP_{jt}	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$	ΔTFP_{jt}	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$
Persistent (β^{η})	0.027***	0.024***	0.046***	0.037***	0.018***	0.082***
$Value ($\beta^{\eta}$)	419	369	688	570	285	1,258
% of Income	1%	1%	1%	1%	0%	2%
B		**High Wage**			**Low Wage**	
Persistent (β^{η})	0.238***	0.239***	0.246***	0.046***	0.030***	0.051***
$Value ($\beta^{\eta}$)	5,874	5,874	6,009	419	285	453
% of Income	7%	7%	7%	1%	1%	1%
C		**Young**			**Old**	
Persistent (β^{η})	0.113***	0.101***	0.144***	0.031***	0.039***	0.021***
$Value ($\beta^{\eta}$)	1,544	1,409	1,896	503	637	335
% of Income	3%	3%	4%	1%	1%	1%

Note: Table 3.4 shows a set of panel regressions controlling for firm and worker characteristics. In each column, the dependent variable is the growth rate of real wages between period $t-1$ and period $t+3$. The main explanatory variable is the persistent component in the firm's productivity process. The row labeled Value (USD) shows the change in real wages resulting from a one standard deviation shock to TFP for a worker with the average wage for that group. The definitions of stayer/switcher, high/low wage and expansion/recession are as in previous tables. $*p < 0.1, **p < 0.05, ***p < 0.01$. Robust standard errors are clustered at the firm level.

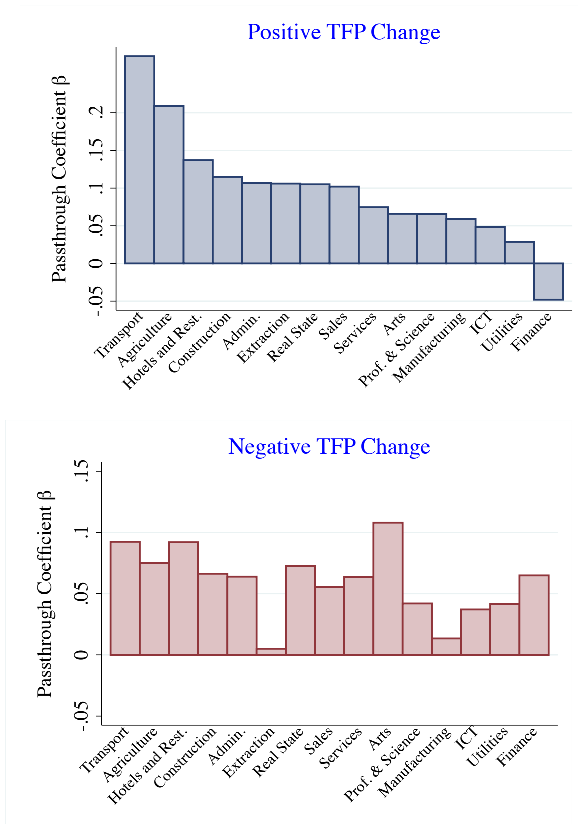
3.5.2 Firm Side Heterogeneity

Does passthrough differ for workers employed by firms in different sectors? Do more productive firms pass a larger or smaller fraction of their productivity gains to wages? To answer the first question we take the sample of continuing and run our passthrough analysis within a set of narrow industry groups.

The left panel of figure 3.5 shows the passthrough coefficient by industry sorted by the magnitude of the coefficient associated with a positive persistent productivity shock. First, notice the marked heterogeneity across sectors. For instance, the passthrough for the Transportation sector is almost ten times the passthrough estimated for Utilities. Second,

process – on long term changes of worker wages although the effects are smaller relative to the persistent shock but still significant. In fact, shocks to the transitory component of productivity have an impact on wages that is half as much as the impact of the persistent component.

Figure 3.5: Passthrough by Industry



Note: The left panel of figure 3.5 shows the coefficient of the change in firm-level productivity on a OLS panel regression as in equation (3.1) conditional on the productivity change to be positive within aggregate industry groups. The right panel shows the results for negative productivity changes. All coefficients are statistically significant at the 1% level with robust standard errors clustered by firm.

Manufacturing sits close to the bottom of the distribution with a coefficient that is close than the economy average. The right panel of figure 3.5 shows the passthrough coefficient for a negative productivity shock. The differences across industries are again remarkable. It is also clear that industries that command high positive passthrough are not the same industries that generate high negative passthrough. Case in point is Finance, which negative passthrough ranks amongst the highest but commands a negative – and not statistically significant – passthrough from positive productivity shocks.

Next, we study whether firms of different productivity levels pass shocks differently to their workers. For instance, it is possible that firms which experience a persistent decline in productivity cannot continue to operate without reducing the wages of their workers. In such cases, we should see a higher passthrough among lower productivity firms. In contrast, if low productivity firms are more vulnerable to productivity shocks due to financial constraints or other frictions, they may leave the market and therefore low TFP firms that stay in the market may show lower passthrough. We defined high productivity firms as those at the top quintile of the TFP level distribution whereas low productivity firms are those in the bottom quintile of the distribution. We then run the same passthrough regression within each productivity group. The results are shown in table 3.5. Comparing columns (1) and (4), one can see that the average passthrough among high TFP firms is one order of magnitude larger than passthrough among low TFP firms. This difference is more stark in the passthrough for negative TFP changes. In fact, the passthrough coefficient of negative shocks is positive and significant for workers in high TFP firms, whereas for workers in low TFP firms, the passthrough is though significant but very small in magnitude. The passthrough of positive shocks is also significantly larger among high TFP firms, though generally smaller than the negative passthrough coefficient. Taken together, these result suggest that low productivity firms provide better insurance to workers in the face of negative shocks. This result is in contrast with the intuition that high TFP firms (which are typically larger and potentially have better access to the financial markets) should be more capable to respond to shocks without impacting labor earnings.

The higher passthrough among high productivity firms could arise from heterogeneous response top shocks between high and low productivity firms or from differences between wage setting.Moscarini and Postel-Vinay (2012) suggests that the employment within large firms, which are typically more productive, are more responsive to shocks than small firms. That is, large firms are quicker to respond to reduce employment growth during a recession to adjust for the lower level of economic activity. Our results indicate that large firms also reduce wages in response to shocks. Dividing the sample further by high wage and low wage workers who work in high TFP and low TFP firms, we see that high wage workers who work at high productivity firms are the most sensitive to persistent TFP shocks.[19]

[19]Our results can also be affected by the selection of firms into or out of the sample. As discussed in the main text, if firms of low TFP are more likely to leave the sample and exit the market, then, the passthrough of negative TFP shocks to wages might be underestimated. Ideally we could use firm borrowing constraints

Table 3.5: Passthrough regression for high and low TFP firms

	(1)	(2)	(3)	(4)	(5)	(6)
	High TFP Firms			**Low TFP Firms**		
	ΔTFP_{jt}	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$	ΔTFP_{jt}	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$
A	**All Workers**					
Persistent (β^η)	0.215***	0.219***	0.177***	0.025***	0.015***	0.075***
$Value ($\beta^\eta$)	$3,088	$3,088	$2,602	$403	$235	$1,175
% of Income	6.5%	6.6%	5.3%	0.8%	0.5%	2.3%
B	**High Wage Workers**					
Persistent (β^η)	0.413***	0.401***	0.240***	0.075***	0.074***	0.138***
$Value ($\beta^\eta$)	$10,339	$9,483	$6,495	$1,846	$1,829	$3,374
% of Income	12.4%	12.0%	7.2%	2.3%	2.2%	4.1%
C	**Low Wage Workers**					
Persistent (β^η)	0.106***	0.097***	0.221***	0.050***	0.034***	0.079***
$Value ($\beta^\eta$)	$906	$822	$1,947	$470	$319	$738
% of Income	3.2%	2.9%	6.6%	1.5%	1.0%	2.4%

Note: Table 3.5 shows a set of panel regressions controlling for firm and worker characteristics. In each column, the dependent variable is the growth rate of real wages for individuals that stay in the same firm for two consecutive periods. The main explanatory variables are estimated shocks to the transitory and persistent components of firm-level (log) TFP. The row named Value (USD) shows the change in real (USD) wages resulting from a one standard deviation shock to TFP for a worker with the average wage for that group. Columns 1-3 show results for firms in the top 20 percentile of the TFP distribution, while columns 4-6 show results for firms in bottom 20 percentile of the TFP distribution. All panels show estimates correcting for worker-side selection but not firm-side selection. $*p < 0.1, **p < 0.05, ***p < 0.01$. Robust standard errors are clustered at the firm level.

3.5.3 Business Cycle Heterogeneity

Like the rest of the world Denmark was hit by a severe economic downturn in 2008. The decline in Danish GDP was under-way at the beginning of 2008 accompanied by a large drop in labor market hiring and an increase in separation rates. Arguably, workers in recessions and expansions face different labor market environments and therefore the passthrough from firm-level idiosyncratic shocks to wages may also be different. To investigate if this is the case, we estimate our passthrough regression separately in recession years (2008 to 2009) and expansion years (2005 to 2006). The results are presented in Table 3.6.

The left three columns are the results for Recessions, and the right three columns show

as an instrument in the selection correction procedure, but we do not currently have that data so firm selection is a potential issue in our analysis

Table 3.6: Passthrough Over the Business Cycle

	(1)	(2)	(3)	(4)	(5)	(6)
	Recessions (08-09)			**Expansions (05-06)**		
	ΔTFP_{jt}	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$	ΔTFP_{jt}	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$
A			**All Workers**			
Persistent (β^{η})	0.083***	0.082***	0.122***	0.036***	0.029***	0.058***
Value (USD)	$1,326	$1,342	$1,879	$570	$453	$890
% of Income	2.5%	2.5%	3.6%	1.0%	0.9%	1.7%
B			**High Wage Workers**			
Persistent (β^{η})	0.181***	0.205***	0.234***	0.091***	0.091***	0.098***
Value (USD)	$4,565	$5,186	$5,790	$2,249	$2,250	$2,433
% of Income	5.4%	6.2%	7.0%	2.7%	2.7%	2.9%
C			**Low Wage Workers**			
Persistent (β^{η})	0.076***	0.052***	0.101***	0.053***	0.044***	0.069***
Value (USD)	$738	$503	$956	$503	$436	$655
% of Income	2.3%	1.6%	3.0%	1.6%	1.3%	2.1%

Note: Table 3.6 shows a set of panel regressions controlling for firm and worker characteristics. In each column, the dependent variable is the growth rate of real wages for individuals that stay in the same firm for two consecutive periods. The main explanatory variable is the change in within-firm (log) TFP. The row named $Value shows the change in real (USD) wages resulting from a one standard deviation shock to TFP for a worker with the average wage for that group. Columns 1-3 show results for years 2008 and 2009, while columns 4-6 show results 2005 and 2006. The top panel shows estimates without correcting for selection while the bottom panel shows selection-corrected estimates. $*p < 0.1, **p < 0.05, ***p < 0.01$. Robust standard errors, below the point estimates, are clustered at the firm level.

the results for Expansions. All panels show the results of estimating passthrough after correcting for selection. The difference between recessions and expansions is clear: in recessions, on average, workers experience significant passthrough when firms experience negative TFP shocks, while in expansions workers are relatively insured against negative shocks – the passthrough for negative TFP shock is significant but small especially for workers between the 20th and 80th wage percentiles.

The results in panel A show the average passthrough effect in expansions vs recessions for all workers. As discussed in the previous subsections, workers with different average wages may be affected differently by TFP shocks. This may be especially true along the business cycle, since potentially low wage workers are more or less sensitive to recessions than high wage workers. The results shown in panels B and C confirm this difference between wage groups. Low wage workers don't have much variation compared to high wage workers in terms of passthrough elasticity, but do see a stronger passthrough effect in recessions compare to expansions. High workers are much more sensitive to recessions. In the recession, high wage worker negative shock passthrough is 10 percentage points higher than it is during an expansion. The difference in the average dollar value is also significant. This is intuitive – firms may be more likely to adjust wages for workers who are paid highly. At the bottom, especially when wages are close to the minimum wage, there isn't much room where firms can adjust wages, so firms are more likely to adjust among other dimensions such as employment or working hours. Generally speaking, low wage workers in expansions are least sensitive to TFP shocks while high wage workers in recessions are the most sensitive. This suggests that low wage workers get much more insurance from the firm when their firms are hit by TFP shocks. High wage workers on the other hand do not have nearly as much insurance, so their wages vary much more due to TFP shocks, especially when the economy is in a downturn.

3.6 Quantitative Analysis

Our previous analysis establishes the key relations between the shocks affecting the firms and the passthrough of these shocks to workers earnings. In this section we take these results as given and study the impact of the passthrough from firms' shocks to workers' wages for inequality and welfare. Doing so is relevant for at least two reasons. First, will allow us to evaluate the social value of the insurance provided by the firms and also to estimate how much workers would be willing to pay in order to increase the degree of insurance they receive. Second, given the large differences in passthrough observed in the data across workers and firms with different characteristics, using a model will allow us to better evaluate the welfare costs of this heterogeneity, the cost of idiosyncratic income fluctuations, and the welfare cost of firm-level shocks.

In order to make progress on these issues, we first estimate a stochastic income process that incorporates both firms' and workers' characteristics. In particular, we extend the standard stochastic income process adopted in the literature by incorporating firm-level shocks that are passed to workers in different degrees. Importantly, we jointly use worker- and firm-level data. Most papers which estimate income processes use only data on individual characteristics, wages, and transitions across different employment statuses (see for instance Low et al. (2010) and Guvenen (2007)). To the best of our knowledge, this is one of the first papers to directly use firm-level shocks and passthrough to wages in the estimation of a stochastic income process allowing for an asymmetric and heterogeneous response of wages to firm shocks.

We then incorporate the estimated income process in an otherwise standard incomplete markets life-cycle consumption-savings model. Using this model we study the welfare value of the (partial) insurance provided by the firms. We do this by means of three counterfactual exercises. In the first, we completely eliminate the insurance provided by the firms (i.e. we allow the passthrough from firm shocks to wages to be equal to one) and we ask what are the welfare losses relative to the baseline, partial insurance, case. In the second, we compare the baseline economy with a case in which the passthrough is zero (i.e. full insurance) and ask how much, in terms of consumption, workers are willing to pay in order to eliminate their exposure to firm-level shocks. In the following sections we set up the basic properties of the stochastic income process and the life cycle income model that we will use in our welfare evaluation.

3.6.1 Wages and Shocks

We assume that the real wage of an individual i working in firm j in period t, $w_{i,j,t}$ is given by,

$$\log w_{i,j,t} = \mu_t + x_{i,j,t}\Gamma + \tilde{\eta}_{i,t} + \tilde{\varepsilon}_{i,t} + \psi_{i,j,t}(z_{j(i),t}, z_{j(i),t-1}), \tag{3.11}$$

where μ_t represents the average level of real wages in the economy, $x_{i,j,t}$ is a vector of regressors including worker and firm level characteristics, $\tilde{\eta}_{i,t}$ is the persistent component of wages that is uncorrelated to firm-level shocks, $\tilde{\varepsilon}_{i,t}$ is the transitory component of wages that is uncorrelated with the persistent component and with firm-level shocks, and $z_{j(i),t}$ is a measure of firm j's TFP which affects all the workers of firm j in period t. Here the subscript $j(i), t$ denotes the firm at which individual i works in period t. The heterogeneity in passthrough is captured by the function $\psi_{i,j,t}$ which may depend on worker and firm-level characteristics.[20]. The function $\psi_{i,j,t}$ is flexible in that it allows for asymmetric responses to increases or decreases in TFP, as well as detailed heterogeneity in the degree of passthrough and the possibility that the worker switches firms between periods $t-1$ and t. As mentioned in section 3.3.3, firm TFP can be further expressed as $z_{j,t} = g(\omega_{j,t-1}) + \eta_{j,t} + \varepsilon_{j,t}$, where $g(\omega_{j,t-1})$ is the anticipated value of productivity, $\eta_{j,t}$ is the unanticipated permanent shock to productivity and $\varepsilon_{j,t}$ is the unanticipated transitory shock.

Estimation Procedure

We start by estimating firms' productivity process. To keep the model as tractable as possible, we assume the TFP follows an AR1 process,

$$z_{j,t} = c_j + \rho_j^z z_{j,t-1} + \xi_{j,t}. \tag{3.12}$$

To obtain the first-order-autocorrelation parameter we demean our estimates of firm-level productivity so $c_j = 0$, and we run the above regression in the data, which gives us a value of ρ^z of 0.97. This value is relatively more persistent than the one used in most of the literature mostly because our TFP estimation carefully controls for observables in a nonparametric fashion as oppose to attributing much of the variation in firm revenues to variation in TFP. We discuss in more detail about our TFP estimation strategy and the corresponding properties in section 3.3.3.[21]. We assume ξ_{jt} is iid and follows a mean zero normal mixture distribution: with probability p, $\xi_{j,t} \sim \mathbb{N}(\mu_1, \sigma_1)$, and with probability $(1-p)$, $\xi_{j,t} \sim \mathbb{N}(\mu_2, \sigma_2)$, where $\mu_2 = -\frac{p}{1-p}\mu 1$. We then estimate the remaining four parameters $\{p, \mu_1, \sigma_1, \sigma_2\}$ following the method developed by Civale et al. (2016). The estimation results are shown in Table 3.7. Note that the moments (mean, variance, skewness, kurtosis) that we use for our estimation in the TFP process are from the worker-weighted TFP distribution. This simplifies the firm-to-worker match since in the current version of our model we do not have multi-worker firms. Each firm only employs one worker and therefore

[20]Notice in this formulation that firm TFP and passthrough parameters enter as an exogenous process. Modeling the endogenous formation of passthrough and decisions of firms in response to exogenous productivity shocks, although beyond the scope of this paper, is an important area of our ongoing research agenda.

[21]Further investigation and discussion on how the TFP measure in this paper compare to the TFP measures in existing literature in terms of properties can be found in Chan et al. (2019)

we match our model to the worker-weighted TFP distribution.

Given our estimated process for firm productivity, we then estimate the worker wage process. In our baseline setting, we assume that firms and workers are randomly matched. Specifically we first draw firms based on the workers weighted distribution (estimated in the previous step). We then draw one worker for each firm, so workers and firms matching is independent. This assumption allows us to search for firm-side parameters and workers-side parameters separately, which greatly simplified the estimation[22]. One of our key innovation is that in our stochastic wage process, we capture wage changes that comes from firms passing TFP shocks to the workers heterogeneously across different worker and firm groups, and non-symmetrically between positive and negative shocks. We assume that the passthrough function, $\psi_{i,j,t}(z_{j(i),t}, z_{j(i),t-1})$, is linear and we ignore the time effect and observable effects for now[23]. The simplified wage process is then:

$$\log y_{i,j,t} = \eta_{i,t} + \varepsilon_{i,t} + \psi_{0i,j,s} z_{j,t-1} + \psi_{1i,j,s}(z_{j,t} - z_{j,t-1})\mathbf{1}_{\Delta z_{j,t}>0} + \psi_{2i,j,s}(z_{j,t} - z_{j,t-1})\mathbf{1}_{\Delta z_{j,t}\leq 0}, \tag{3.13}$$

where $\eta_{i,t}$ is the permanent component of workers wages which follows a standard AR(1) process: $\eta_{i,t} = c_i + \rho_i^\eta \eta_{i,t-1} + \zeta_{i,t}$, and $\varepsilon_{i,t}$ is a transitory component. We assume that $\zeta(i,t) \sim \mathbb{N}(0, \sigma_\zeta)$ and $\varepsilon_{i,t} \sim \mathbb{N}(0, \sigma_\epsilon)$, so we have now seven parameters to estimate:

$$\omega = \{\sigma_i^\varepsilon, \sigma_i^\zeta, c_i, \rho_i^\eta, \psi_0, \psi_1, \psi_2\}.$$

[22]Alternatively, one could draw firms and workers jointly according to the joint distribution that we observe in the data. This will then require the joint estimation of all firms side parameters and workers side parameters as well as the covariance parameters.

[23]Time effect and workers observable characteristics are added in the full model section, and the estimation of the full model is ongoing work.

The wage process specified in equation 3.13 implies the following moments,

$$\begin{aligned}
\Delta \log y_{ijt} &= \Delta\eta_{i,t} + \Delta\varepsilon_{it} + (\psi_1 \mathbf{1}_{\Delta z_{j,t}>0} + \psi_2 \mathbf{1}_{\Delta z_{j,t}\leq 0})\Delta z_{j,t} \\
&\quad + (\psi_0 - \psi_1 \mathbf{1}_{\Delta z_{j,t-1}>0} - \psi_2 \mathbf{1}_{\Delta z_{j,t-1}\leq 0})\Delta z_{j,t-1} \\
\mathbb{E}(\log y_{ijt}) &= \frac{c_i}{1-\rho_i^{\eta}} + F_1(\psi_0, \psi_1, \psi_2, \text{firmparams}) \\
Sd(\log y_{ijt}) &= \sigma_i^{\varepsilon} + \frac{\sigma_i^{\zeta}}{\sqrt{1-\rho_i^{\eta 2}}} + F_2(\psi_0, \psi_1, \psi_2, \text{firmparams}) \\
\mathbb{E}(\log y_{ijt} \log y_{ijt-1}) &= \frac{c_i^2}{1-\rho_\eta} + \rho_\eta \frac{\sigma_\zeta^2}{1-\rho_\eta^2} + F_3(\psi_0, \psi_1, \psi_2, \text{firmparams}) \\
Sd(\Delta \log y_{ijt,t-1}) &= \sqrt{2}\sigma_i^{\varepsilon} + \sqrt{\frac{2}{1+\rho}}\sigma_i^{\zeta} + F_4(\psi_0, \psi_1, \psi_2, \text{firmparams}) \\
Sd(\Delta \log y_{ijt,t-3}) &= \sqrt{2}\sigma_i^{\varepsilon} + \sqrt{\frac{2}{1+\rho}}(1+\rho+\rho^2)\sigma_i^{\zeta} + F_5(\psi_0, \psi_1, \psi_2, \text{firmparams}).
\end{aligned}$$

$F_1 - F_5$ are functions of ψ_0, ψ_1, ψ_2, firm parameters, and TFP values which are predetermined in the firm side estimation. Given the asymmetric passthrough structure of our wage process, it is rather difficult to derive an analysis solution for our parameters. Hence, we instead use a mixture of simulated method of moments (SMM) and indirect inference (Smith Jr (1993)) to jointly estimate all seven parameters. Note that the mean and variance of log wage, the variance of the change of log wages at one and three years, and the one-period autocorrelation of wages gives us information about the first four parameters. To identify the three passthrough parameters, consider the auxiliary models:

$$\log y_{ijt} = \bar{\beta} + \beta_0 z_{j,t-1} + \epsilon_1 \tag{3.14}$$

$$\Delta \log y_{ijt} = \bar{\gamma} + \gamma_1 \Delta z_{j,t} \mathbf{1}_{\Delta z_{j,t}>0} + \gamma_2 \Delta z_{j,t} \mathbf{1}_{\Delta z_{j,t}\leq 0} + \epsilon_2. \tag{3.15}$$

The goal is to bring the data and simulated data as close as possible through the lens of auxiliary model. That is, given a set of parameter guesses, we run the regression of the auxiliary model using the data (which gives us $\hat{\beta}_0$, $\hat{\gamma}_1$ and $\hat{\gamma}_2$) and using the simulated data generated from our economic model (which gives us $\tilde{\beta}_0$, $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$). We want to bring ($\hat{\beta}_0$, $\hat{\gamma}_1$, $\hat{\gamma}_2$) and ($\tilde{\beta}_0$, $\tilde{\gamma}_1$, $\tilde{\gamma}_2$) as close as possible. Matching this auxiliary model's moments will give us information about ψ_0, ψ_1 and ψ_2. All together we have seven parameters to be

Table 3.7: TFP and Wage Parameter Estimation

TFP Parameters		
Variable	Value	Description
ρ	0.9702	AR(1) parameter
p	0.9676	Probability of the normal mixture of innovation in TFP
μ_1	0.1409	mean of the first normal distribution in innovation
σ_1	0.0017	Standard deviation of the first normal distribution in innovation
σ_2	0.7382	Standard deviation of the second normal distribution in innovation
Wage Process Parameters		
Variable	Value	Description
ρ_η	0.701	AR(1) parameter
σ_η	0.1884	Standard deviation of the permanent wage shock
σ_ε	0.0537	Standard deviation of the transitory wage shock
c_η	3.8	Average component in wage process
ψ_0	0.0446	TFP marginal effect on wage levels
ψ_1	0.0126	Positive TFP shock marginal effect on wage levels
ψ_2	0.0036	Negative TFP shock marginal effect on wage levels

identified, and we are matching the following eight moments to identify the parameters of the wage process:

$$Moments = \{\mathbb{E}(\log y_{ijt}), \mathbb{E}(\log y_{ijt} \log y_{ijt-1}), Sd(\log y_{ijt}), Sd(\Delta \log y_{ijt}), Sd(\Delta \log y_{ijt,t-3}), \hat{\beta}_0, \hat{\gamma}_1, \hat{\gamma}_2\}.$$

This simple way of estimation will give us the estimation of the seven parameters for the wage process ($\omega_1 - \omega_7$).

3.6.2 Estimation Results

We further simplify the estimation considering that workers and firms are ex-ante homogeneous. This reduces the number of parameters to be estimated to twelve (five for firms and seven for workers).[24] The estimation results are shown in Table 3.7, and Table 3.8 shows the model fit.

[24]Our current work extends our estimation method to account for fixed different across different firms and worker types.

Table 3.8: Model Estimation Fit

TFP Moments			
Variable	Data	Model	Description
$\mathbb{E}(z)$	0	0.0001	Mean of log TFP process parameter
$V(z)$	14.34	10.35	Variance of log TFP process
$S(z)$	-1.02	-0.91	Skewness of log TFP process
$K(z)$	3.32	3.89	Kurtosis of log TFP process
Wage Moments			
Variable	Data	Model	Description
$\mathbb{E}(y)$	12.74	12.74	Mean log wage (DKK)
$Sd(y)$	0.15	0.07	Standard deviation of log wage
$\mathbb{E}(y * y_L)$	162.28	162.27	Autocorrelation of log wage
$Sd(\Delta(y))$	0.03	0.05	Standard deviation of changes in log wage
$Sd(\Delta_3(y))$	0.09	0.09	Standard deviation of changes in 3 periods log wage
$\hat{\beta}_0$	0.04	0.04	Auxiliary model coefficient
$\hat{\gamma}_1$	0.009	0.009	Auxiliary model coefficient
$\hat{\gamma}_2$	0.009	0.009	Auxiliary model coefficient

3.6.3 A Consumption-Savings Model

In this section we study some key implications of the rich earnings dynamics generated by the stochastic process estimated in the previous section. We pose an heterogeneous agents incomplete markets life-cycle model in which workers are subject to the stochastic process described by equation 3.11. Individuals can borrow and save using a risk-free asset, $a_{i,t}$, with gross return $(1 + r_t)$. Borrowing is limited by a predefined minimum level which in principle can depend on worker characteristics. Denote this minimum value as $\underline{a}_{i,t}$. We will also assume that the individuals pay taxes and receive benefits from the government, which will be modeled to match the Danish system. Finally, individuals value consumption $c_{i,t}$ by means of a time separable utility function, $u(c_{i,t})$. The dynamic programming problem of an individual is given by,

$$V_t^i(a_{i,t}, w_{i,j,t}, \mu_t) = \max_{c_{i,t}, a_{i,t+1}} \left\{ \frac{(c_t^i)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} V_t^i(a_{i,t+1}, w_{i,j,t+1}, \mu_{t+1}) \right\},$$

$$\text{subject to}$$

$$c_{i,t} + a_{i,t+1} = a_{i,t}(1 + r_t) + (1 - \tau_t(w_{i,j,t}))w_{i,j,t} + T_t(w_{i,j,t}),$$

$$\log w_{i,j,t} = \mu_t + \tilde{\eta}_{i,t} + \tilde{\varepsilon}_{i,t} + \psi_{i,j,t}(z_{j(i),t}, z_{j(i),t-1}),$$

$$\mu_{t+1} = \Gamma(\mu_t), a_{i,t+1} \geq \underline{a}$$

where β is the subjective discount factor, σ governs risk aversion and the intertemporal elasticity of substitution, and μ_t is the distribution of individuals over idiosyncratic states. The tax function is defined by $\tau_t(w_{i,j,t})$ while $T_t(w_{i,j,t})$ is a government benefits function.

Calibration

To simplify the analysis, we assume that individuals are infinitely lived, and a period in our model corresponds to a year. The coefficient of risk aversion σ is set to 2 as a conservative choice. The discount factor is chosen to match a wealth-to-income ratio of 4, the returns to the risk free asset is set to 3%, and the borrowing limit, $\underline{a}$ is set equal to the average annual earnings in the economy.

The key element of our analysis is the stochastic income process faced by workers, and more importantly, the passthrough of firms' shocks to workers' wages. In our baseline analysis we consider the income process described by equation 3.13 and the parameters from table 3.7. We then consider two cases, the first sets $\psi_1 = \psi_2 = 1$ which corresponds to a full symmetric passthrough of productivity shocks to wages. The second turns off the passthrough of positive shocks by setting $\psi_1 = 1$ but $\psi_2 = 1$.

Model Fit

Our model economy is able to generate substantial wealth inequality. Estimates from Jakobsen et al. (2017) show that in Denmark, 20% of total wealth is held by households at the top 1% of the wealth distribution, whereas 50% of total wealth is held by the top 10%. The bottom 50% of the distribution holds little to no wealth. In our model, the top 1%

holds 15% of the wealth in the economy whereas the top 10% of households holds 45% of the total wealth. This is remarkable considering that standard consumption-savings models typically cannot account well for the large disparities in wealth observed in Denmark or other economies.[25]

Our first quantitative exercise allows answering the question: what is the value – in consumption equivalents – that workers assign to the insurance provided by the firms? In our simple setting, this implies comparing the benchmark economy to one in which firms' shocks are fully passed to workers ($\psi = \psi = 1$). Our estimates suggest that workers are willing to pay a very little amount of lifetime consumption for the insurance provided by firms (less than 1%). This is because, in a model with infinitely lived workers with access to a risk-free asset, households can undue the decrease of the insurance provided by the firms by increasing their life-time savings. The offsetting effect of an increase in capital accumulation reduces the steady-state value of the insurance provided by firms. At the same time, because there is an increase in the fraction of positive shocks that are passed to workers, the average workers wage increases. The value of the insurance for negative shocks, as expected, is more valuable for workers. We can evaluate that case by considering a 0 passthrough of positive shock, but a full passthrough from negative firms' shocks to workers wages. Adding a more realistic labor income process that takes into account employment status transition and a richer asset market, both of which are part of our ongoing work, will likely increase the insurance value provided by the firms.

3.7 Conclusion

In this paper, we offer new evidence on the effect of changes in firms' productivity on workers' wages. Using high quality, employer-employee matched administrative panel data we address two important issues the literature has ignored so far: the effect of selection and the impact of changes in firm-level productivity for workers that switch between firms. Moreover, we provide a more direct measure of firm' total factor productivity and we explore several degrees of heterogeneity among firms and workers types.

To control for selection, we use a novel approach that exploits employment and income

[25] For a survey see De Nardi (2015) and the references therein.

information of spouses to estimate the probability that an individual stays in the same firm during a particular year. We find that controlling for selection has a major impact in the passthrough estimates from TFP shocks to wages. In fact, the OLS coefficient more than doubles when one has controlled for selection relative to the coefficient when selection issues are not addressed. In general, we find large and economically significant passthrough coefficients: After we have controlled for selection, we find that a worker in a firm that experiences a TFP change of one standard deviation sees her annual earnings increase by $1,500 which is around 2% of the Danish income per capita. Considering that in any given year 33% of firms - which employ 40% of all the workers in Denmark - receive experience a persistent or transitory TFP shock of at least one standard deviation from the average, we see that the effect of firm-level shocks on wages is quite substantial. Furthermore, relative to continuing workers, the impact of TFP change for switchers is substantially larger.

Heterogeneity plays a major effect on shaping the effect of TFP shocks on wages. In fact, we find remarkable differences between workers at higher ranks of the income distribution – who are less insured against the positive and negative shocks affecting the firms – and workers at the bottom of the income distribution – for which the passthrough is lower and less economically significant. We also find extremely large differences across industries and for young and old workers.

In the second part of our paper, we estimate a rich stochastic income process that captures the salient features we observe in the data. Our major innovation is to consider an independent process of firm-level productivity as an additional shock affecting workers earnings directly and the estimation of a passthrough from the firm's shocks to workers wages using indirect inference. Our estimation suggests an important role for firm-level shocks in shaping the dynamics of workers' labor income. We then incorporate our estimated stochastic process into an otherwise standard consumption savings model. In our model, the insurance provided by the firms is of little value for the workers which can offset an increase in the passthrough from firm's shocks to wages – which increases income instability – by increasing asset accumulation. Incorporating a richer life-cycle into the model and a more realistic asset market will likely increase the importance of the insurance provided by firms, both of which are part of our ongoing work.

Chapter 4

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Appendix A

Appendix to Chapter 1

A.1 Data Appendix

A.1.1 The PSID Sample

The PSID sample used for studying the time series of the share of entrepreneurs and other statistics used in this study was constructed as follows. From the raw data, I extract a sample of heads of household from the SRC sample (I do not consider information of the SEO, Immigrants, or Latino Sample) from the waves going from 1970 and 2015. Some individuals have missing observations in employment status or were registered as "refusing to answer". In those cases I replace the employment status variable by the code corresponding to "no working for money" (code 3). Only 107 observations were replaced in this way. The variable defining the age of the head of the household has several inconsistencies that are necessary to fix. In particular, for those individuals whose age jumps up for more than 3 years, or jumps down, I imputed an increase in age based on the first reliable age. Similar to age, the education variables has many inconsistencies. Because in this paper I focus on the education as a measure of skill, I create a new variable that considers the highest educational attainment of the individual as a measure of education. All monetary variables (income, wealth, etc.) were deflated using the Personal Consumption Expenditure index from the Bureau of Economic Analysis. The baseline sample considers households whose head is between 22 and 60 years old, both ends included. This yields a sample of 112,283 year-household observations with an average of 3,118 observations per year. For the period in which I focus my study, that is 1985 to 2015, the number of observations is 75,031 with an average of 5,573 observations per year. All statistics were calculated using PSID sample weights.

To calculate the parameters of the income process of equation (1.9) I measure earnings as the real value of total labor income of individual i in period t. Total labor includes wages and salaries, tips, commissions, and bonuses. However, the results are quite similar if one uses wages and salaries as measure of labor income. Then, I drop all observations of individuals that are self-employed business owners in either period t or $t-1$. Given these restrictions, the parameters of equation (1.9) are calculated on a sample of 6,303 individuals and 58,094 observations. The value of ρ_w is 0.73 with and standard error of .0028. The adjusted R^2 of the regression is 0.55. The standard error of the residuals 0.53 so I set σ_y to this value. Estimating equation (1.9) only for individuals with college or more generates a slightly lower value for ρ_w equal to 0.70 (0.72 for individuals with high school or less). Nevertheless, the standard error of the residuals is quite similar and equal to 0.53.

A.1.2 CPS Data, Supply of Skills and College Premium

In this section, I describe how I constructed the share of college graduates (which is equated to the share of high skill workers in the model) and the college premium (which is the skill premium in the model). To calculate both time series I draw a sample of individuals from the March CPS data (accessed through IPUMS) from 1970 to 2015. To keep the sample selection as close as possible to the PSID, I keep individuals that are head of the household aged between 22 and 60 years (both ends included) which are in the labor force and have valid education information. Individuals in the armed force or with negative weights are also excluded from the sample. The baseline sample consists of 1.7 million individual-year observations.

The share of high skill workers is the weighted proportion of individuals with a college degree or more. The center panel of figure 1.8 shows the corresponding time series from 1980 to 2015. The skill premium, on the other hand, is calculated over a subsample of wage workers only. This is because both in the model and in the literature, the skill premium is the relative wage of high skill workers to low skill workers. To avoid issues related to the differences in labor supply of college graduates versus non-college graduated, here I consider a sample of individuals wage workers (not self-employed) that worked more than 40 weeks, and more than 35 hours per week (which is the definition of full time workers in Acemoglu and Autor (2011)). This leave yields a sample of 1.2 million observations. Then, the college premium is the difference between the weighted average of log-real wage for college graduates and the weighted average of log-real wage for high school graduates. The right panel of figure 1.8 shows the corresponding time series from 1985 to 2015.

A.1.3 Aggregates

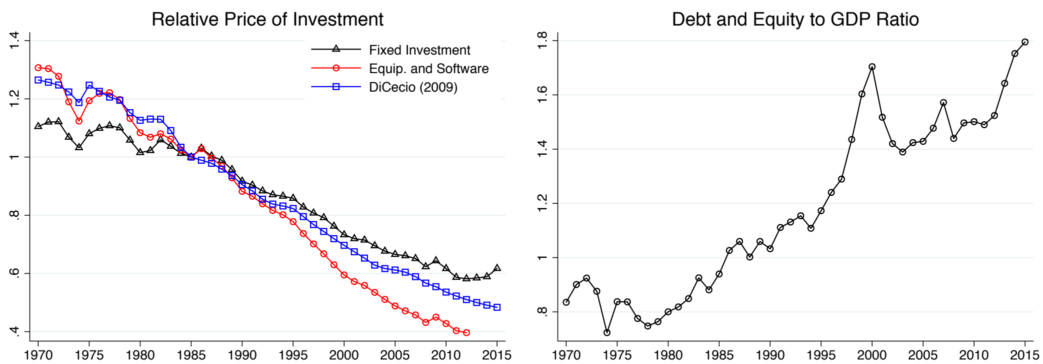
In this section, I show some additional details on the construction of the relative price of capital goods used in my quantitative exercise and the measure of debt and equity relative to non-financial private sector GDP.

I take the measure of the relative price of investment directly from DiCecio (2009) estimates available in the Federal Reserve Bank of St. Louis website (FRED time series PIRIC). Alternatively, one could calculate the relative price of investment as the ratio of the price index of non residential investment calculated by the BEA (FRED time series A008RD3Q086SBEA) divided by the price index of non durable consumption (FRED series CUUR0000SAN). A third alternative is to use a more refined measure of investment that only considers equipment and software (FRED time series A010RD3A086NBEA) relative to the price index of non durable consumption. The left panel of figure A.1 shows the evolution of each of these series re scaled to 1985. The three time series show a similar declining pattern although measure of the price of investment that considers equipment and software only shows a sharper decline: relative to 1985 the measure that considers all investment declined 40%, DiCecio (2009) measure declined a 55%, and the measure that considers equipment and software declined 60%. This makes the choice of the quality adjusted time series calculated by DiCecio (2009)'s a conservative option, right in the middle of these different measures.

To the ratio of debt and equity to business GDP that serves as one of the targets in my quantitative exercise I consider four different time series. From the Flow of Funds I consider the Non Financial Non Corporate Businesses Total Liabilities (FRED time series NNBTILQ027S) and the Non Financial Corporate Businesses Total Liabilities and Equity

(FRED time series NCBLEYQ027S). Both series are aggregated averaging the quarterly data to annual levels. Then I add these annual series to have a measure of the total liabilities of the non financial business sector. The measure of GDP comes from BEA sectoral measures of GDP from which I add up the annual nominal GDP across all private industries with the exception of Finance and Insurance. The right panel of figure A.1 shows the resulting time series.

Figure A.1: PRICE OF INVESTMENT AND DEBT-TO-GDP RATIO



Note: The left panel of figure A.1 shows the time series of the relative price of investment for three different measures. The right panel shows the debt and equity of non financial businesses to non financial businesses GDP ratio.

A.2 The Algorithm

The solution of the model implies calculating an initial and final steady states, and the complete transition path of aggregate states and factor prices, given the exogenous sequences of $p_{k,t}$, H_t, and $A_{H,t}$. On top of the well known complications of solving heterogenous agents models, the present model requires finding a combination of three prices that simultaneously clear the markets for capital, high skill labor, and low skill labor.

To solve the steady states of the economy I proceed as follows. Consider that the economy is at the steady state in $t = 0$ and $p_{k,t} = p_{ss,0} = 1$, $H_t = \overline{H}$, and $A_{H,t} = 1$. Individuals expect this vector of aggregate states variables to remain constant forever. Then, given this aggregate vector, the algorithm to find the stationary equilibrium is as follows,

- S0: Guess a vector of prices $\{\tilde{\omega}_H, \tilde{\omega}_L, \tilde{r}\}$ and solve the problem of the households in 1.2, that is, solve the profit maximization problem of the entrepreneurs, and get the corresponding policy rules and factor demands for the households. To solve the problem of the households I use Value Function Iteration searchings continuously over the asset space.

- S1: Given an initial distribution of individuals over idiosyncratic states, μ_t, iterate until convergence and calculate the aggregate demand of capital, high skill labor, and low skill skilled labor coming from the entrepreneurs. Denote these by $K^{D,e}, N_H^{D,e}$, and $N_L^{D,e}$ respectively. Calculate also the aggregate supplies of capital, and each type of labor, K^S, N_H^S, and N_L^S.
- S2: Calculate the demands of high and low skilled labor of the non-entrepreneurial sector as the residual supply after subtracting the demands for the entrepreneurial sector, that is $N_H = N_H^S - N_H^{D,e}$ and $N_L = N_L^S - N_L^{D,e}$. If these result in negative supplies, go to S0 and guess a new set of prices with a larger value of the wages.
- S3: If the residuals demands of labor are positive in S2, use the first order condition of the problem of the non-entrepreneurial sector to find K, that is, solve the nonlinear expression, $p_{ss,0}(\tilde{r} + \delta) - F_K\left(N_{H,}, N_L, K^C\right) = 0$.
- S4: Using N_H and N_L from S2 and K^C from S3, check the three following conditions,
  - $\tilde{\omega}_H - F_H(N_H, N_L, K)$,
  - $\tilde{\omega}_L - F_L(N_H, N_L, K)$,
  - $K^S - K^C + K^{D,e}$

  If the sum of the last three expressions is greater than 10^{-6}, go to $S0$ using a new guess of prices. If not, the equilibrium set of policy functions, value functions, prices, and stationary distribution of individuals over idiosyncratic states has been found.

Notice that we could calculate in S2 the residual demand of capital for the corporate sector, and go directly to S4 to check $\tilde{r} = F_K\left(N_{H,}, N_L, K^C\right) - \delta$. In practice, I have found that the previous algorithm is much more stable since it avoids iterating over the interest rate. I repeat these same steps both for the initial and final steady states, changing only the value of the aggregates, $p_{k,t}$, $A_{H,t}$, and H_t.

To calculate the transition path of the economy between the initial and final steady states requires taking a stand of what do the household know about the evolution of the economy from period 0 to the infinite future. Here, we can take two extremes cases. One can assume that individuals have perfect foresight about the full equilibrium path of prices and aggregate states, or one can assume that individuals are myopic in the sense that they are surprised by the change of the relative price of investment goods and perceive that such price will remain fixed forever. Here I describe both algorithms in detail.

Perfect Foresight Case. Given a sequence of aggregate states $\Theta_t = \{p_{k,t}, A_{H,t}, H_t\}_{t=0}^{T}$ and a fixed value of the vector after T periods, $\Theta_T = \{p_{k,T}, A_{H,T}, H_T\}$ for all $t > T$, I proceed as follows,

- P0: take $\Theta_t = \Theta_0$ and $\Theta_t = \Theta_T$ and calculate the corresponding stationary equilibrium recording the equilibrium prices, and value functions. Denote the stationary distribution of the first steady state as $\mu_{ss,1}$.
- P1: Guess a path of prices $\{\tilde{\omega}_{H,t}, \tilde{\omega}_{L,t}, \tilde{r}_t\}_{t=1}^{t=T}$ which is fully observed by the agents at the end of period $t = 0$,
- P2: Starting in period $T-1$, take the continuation values of the problem of the households as given and solve

$$V_{T-1}^{s}\left(a_{T-1}, z_{T-1}, y_{T-1}, d_{T-2}\right) = \max_{c_{T-1}, a_T} \left\{ \frac{c_{T-1}^{1-\sigma}}{1-\sigma} + \beta \left[\chi \mathbb{E}_{z'|z, y'|y} V_T^s\left(a_T, z_T, y_T, e_T\right) + (1-\chi) \sum_{j \in \{H,T\}} \zeta_{s,j} \mathbb{E} V_T^s\left(a_T, z_T, y_T, e_T\right) \right] \right\}$$

$$c_{T-1} + a_{T-1} \leq \left(1 + \tilde{r}_{T-1}\right) a_{T-1} + \pi_s\left(z_{T-1}, a_{T-1}\right) - \mathbb{I}\left(d_{T-2} = w\right) \kappa,$$

  on a grid of $a's, z's$, and $y'z$. Do the same for workers, and record the value functions, V_{T-1}^s.
- P3: Go to period $T-2$, take V_{T-1}^s as given, and solve the problem entrepreneurs and workers in $T-2$ recording the continuation values. Continue until $t = 1$.

This generates a path of value functions that are consistent with $\{\tilde{\omega}_{H,t}, \tilde{\omega}_{L,t}, \tilde{r}_t\}_{t=1}^{t=T}$. Notice, however, that *these are not the equilibrium prices.* To find the equilibrium prices now we need to iterate forward, taking the initial distribution as given, and solving for the equilibrium prices in every period. Notice that in going forward, we shall not use the guessed set of prices. To iterate forward, I proceed as follows.

- F1: Given $\mu_0 = \mu_{ss,1}$ and the continuation values, V_1^s for $s = \{H, L\}$, solve for a new set of prices $\{\omega_{1,H}, \omega_{1,L}, r_1\}$ that clears the markets and record the resulting μ_1 *without using the guessed sequence of prices*

- F2: To solve the equilibrium in a given period

  - (A) Guess $\{\hat{\omega}_{1,H}, \hat{\omega}_{1,L}, \hat{r}_1\}$ and solve the problem of the agents
  - (B) Given μ_0 and the policy functions, calculate the excess demand and check market clearing as in S4
  - (C) If prices clear the markets (tolerance 10^{-5}) stop, record the new equilibrium prices and the results distribution, μ_2 and go to the next period,
  - (D) Otherwise, guess a new set of prices and go to (A)

- F3: Proceed in the same way until period T to generate a new path of *equilibrium prices,* $\{\omega_{t,H}, \omega_{t,L}, r_t\}_{t=1}^{t=T}$. Compare them with $\{\tilde{\omega}_{t,H}, \tilde{\omega}_{t,L}, \tilde{r}_t\}_{t=1}^{t=T}$ if the maximum distance is greater than 10^{-4}, take a weighted average of the series as a new guess and go to S1, stop other wise

Upon completion, we have found a path of prices, continuation values, policy function, and distributions that are consistent with the equilibrium along the time series of $\{\Theta_t\}_{t=0}^{T}$. Given that the algorithm needs to find a vector of three prices in each period which is consistent with market clearing it requires very good initial conditions. I have found that the standard method of starting with a linear trend of prices between the initial and last steady states works quite poorly. Instead, calculating the stationary equilibrium for several points on the path of $\{\Theta_t\}_{t=0}^{T}$ and using a linear path between these points ensure a faster, more accurate, solution.

Myopic Case. Alternatively, we can assume that individuals are surprised every period by the changes of the exogenous process of Θ_t and every time they see a new aggregate vector, they perceive this as remain fixed for the infinite future. To solve the transition in this case I proceed as follows.

- M1: Solve the initial steady state of the economy with $\Theta_t = \Theta_0$ and save the steady state distribution, μ_0
- M2: Go to period $t = 1$ with Θ_1 and assume that individuals think that $\Theta_t = \Theta_1$ for all t, and solve for the equilibrium prices as follows,
  - Guess a vector of $\{\hat{\omega}_{1,H}, \hat{\omega}_{1,L}, \hat{r}_1\}$, solve the household's problem, and record the policy functions. Here we can use value function iteration because future is perceived as "the same" by the agents.
  - Taking μ_0 and given and the policy functions, check the equilibrium conditions for capital and labor as in S4 above. If they hold, then we have found the equilibrium prices. If not, guess a new set of prices.
- M3: When the prices have been found, update μ_0 to μ_1 and
- M4: Go to period $t = 2$ and start again in point M2, and proceed until the entire transition path is completed.

This generates a new path of equilibrium prices and a distribution of agents over idiosyncratic states.

Some additional details on the numerical implementation of the model are important. The problem of the households is large and contains several state variables that one needs to keep track of. To maintain tractability I choose a coarse grid of 7 points in the labor productivity, y, and a denser grid of 11 points for the entrepreneurial ability, z. Both stochastic processes are discretized using a modified version of the Tauchen (1986) method. In the particular case of z, since most of the action in terms of switching between occupations happen at the upper end of the distribution of z, I place more point in that area of the grid. In other words, I do not choose a equally spaced grid for the z process. Finally, for the grid of assets, I choose a corse grid of 205 points. Because the Value Function has kinks at the points of occupational switching, I solve the problem using Value Function Iteration to ensure the accuracy of the solution. For the same reason, I solve the equilibrium of the model simulating the PDF of the distribution of individuals over the idiosyncratic distribution. The main challenge of solving the model is finding the vector of prices that clear the labor and capital markets. There is no clear guidance to solve such non linear system of equations that involves aggregating the individuals' decisions. Consequently, I trade accuracy for speed. I found that solving first the steady state in each of the transition gives excellent initial conditions for solving the problem along the transition path.

A.3 Model extensions

In this section I consider several extensions and modifications of the baseline version of the model.

A.3.1 Collateral constraint

In the baseline version of my model entrepreneurs require working capital to run their firms and they can borrow resources up to a fraction $\lambda > 1$ of their own wealth, a. This assumption was made to have an interesting policy comparison. In this section I consider a more standard assumption. Entrepreneurs are not required to pay workers in advance and can borrow capital up to λa. The problem of the workers does not change, but the problem of the entrepreneurs changes to

$$V^e_{s,t}\left(\Omega_t, \Theta_t, \mu_t\right) = \max_{c_t, a_{t+1}} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \left[\chi \mathbb{E}_{z_{t+1}|z_t, y_{t+1}|y_t} V_{s,t+1}\left(\Omega_{t+1}, \Theta_{t+1}, \mu_{t+1}\right) + \right.\right.$$
$$\left.\left. \left(1-\chi\right)\eta \sum_{j\in\{H,L\}} \zeta_{s,j} \mathbb{E} V_{s,t+1}\left(\Omega_{t+1}, \Theta_{t+1}, \mu_{t+1}\right) \right] \right\}, \quad \text{(A.1)}$$

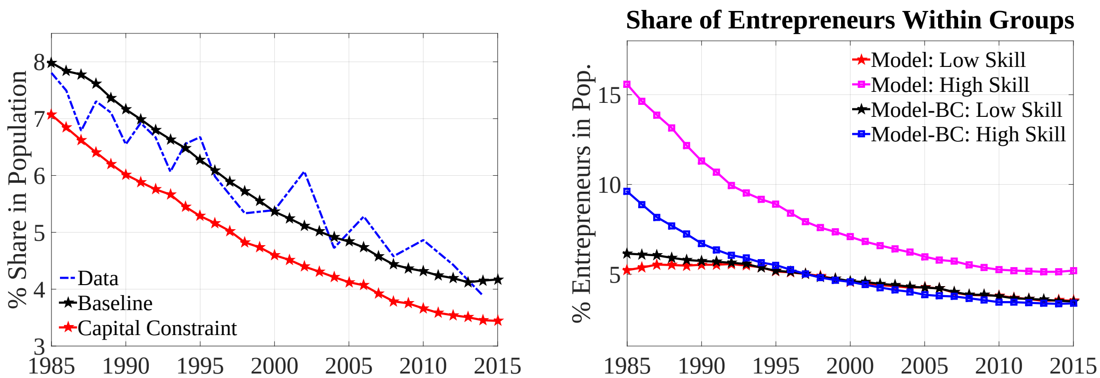
$$\pi_{s,t}(z_t, a_t) = \max_{n_{H,t}, n_{L,t}, k_t} \Bigg\{ z_t \theta_s \left[f\left(n_{H,t}, n_{L,t}, k_t\right)\right]^{\gamma} - p_{k,t}\left(r+\delta\right) k_t - \\ \omega_{H,t}\left(\Theta_t, \mu_t\right) n_{H,t} - \omega_{L,t}\left(\Theta_t, \mu_t\right) n_{L,t} \Bigg\},$$

$$c_t + a_{t+1} \leq \left(1 + r\left(\Theta_t, \mu_t\right)\right) a_t + \pi_{s,t}\left(z_t, a_t\right) - \mathbb{I}\left(d_{t-1} = w_{t-1}\right)\kappa,$$
$$p_{k,t} k_t \leq \lambda a_t, a_{t+1} \geq 0.$$

The first thing to notice is that written in this, the equilibrium of this economy is constrained efficient. Notice this is not the case of the original problem because wages appear in the borrowing constraint. In such case the decentralized equilibrium does not necessarily coincide with planner's solution as the latter internalized the fact that choosing a different allocation, it relaxes the borrowing constraint, moving the economy closer to the efficient optimum. This is not the case with the problem in equation A.1 as the value of $p_{k,t}$ is assumed to be exogenous and therefore is not part of the choice of the planner.

The second thing to notice is that the borrowing constraint relaxes over time as $p_{k,t}$ decreases. Everything else equal, a decrease in the relative price of capital would induce more individuals to start a firm, increasing the share of entrepreneurs in the economy. Still, because of the general equilibrium effects and the effects of $A_{H,t}$ and H_t, the decrease in the share of entrepreneurs is quite similar to my baseline results in section 1.4. The left panel of figure A.2 shows decline almost does not change in this case relative to the data an and the baseline results. However, because I am using the same set of parameters used in my baseline exercise, the level of the share of entrepreneurs is lower in this case. The right panel shows that the distribution within skill groups changes substantially. Notice also that the share of entrepreneurs declines in this case. This is because, with a more relaxed borrowing constraint, entrepreneurs can operate even closer to their optimal level. General equilibrium will increase even more the wages, reducing the share of entrepreneurs in the economy.

Figure A.2: Evolution of the Share of Entrepreneurs



Note: The left panel of figure A.2 shows the evolution of the share of entrepreneurs as calculated from the PSID data (blue dashed line) and the share of entrepreneurs implied by the baseline model (black-starred line) and by the model with a more standard borrowing constraint $p_{k,t}k_t \leq \lambda a_t$. The right panel shows the share of entrepreneurs within skill group implied by the model. Lines labeled as Baseline correspond to the basic results. Lines labelled as Model-BC consider the modified borrowing constraint.

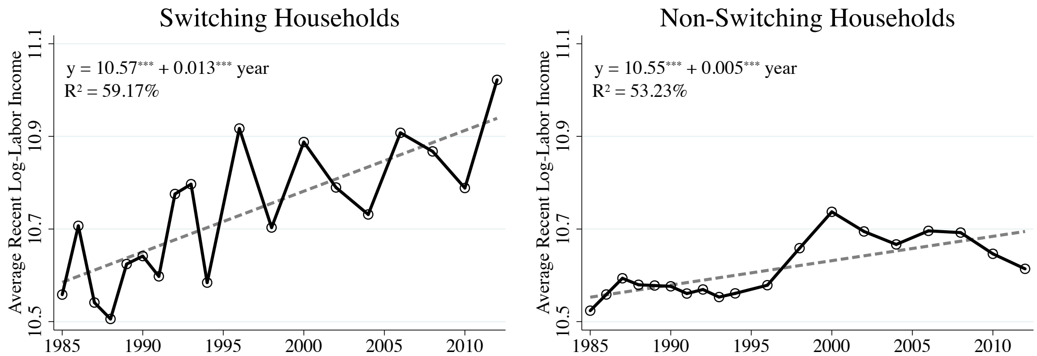
A.4 Appendix Figures

Table A.1: Sample Characteristics

	Wage Workers	Bus. Owners	Active Bus. Owners	Self-Emp. Bus. Owners	Entrepreneurs
Num. obs. per year	2,922	609	526	372	203
Fam. Income (M)	69.2	123.3	124.4	134.8	161.2
Age (mean)	39.8	43.1	43.2	44.0	44.3
Males (%)	70.7	89.7	90.0	91.7	92.9
Drop Outs (%)	7.4	2.8	2.8	2.8	1.4
High School (%)	31.1	24.3	25.0	25.9	18.3
Some College (%)	26.4	25.4	25.9	25.5	22.4
College and More (%)	35.2	47.5	46.4	45.8	57.9
White (%)	87.2	95.5	95.6	96.0	96.3
10th Pct. Wealth (M)	-7.8	5.5	6,252	15249	39.2
50th Pct. Wealth (M)	39.0	267.0	278.9	358,9	493.0
90th Pct. Wealth (M)	414.2	1,605.0	1,729.2	2,068.5	2,601.3
95th Pct. Wealth (M)	684.9	2,750.0	2,959.4	3,497.8	4,326.6

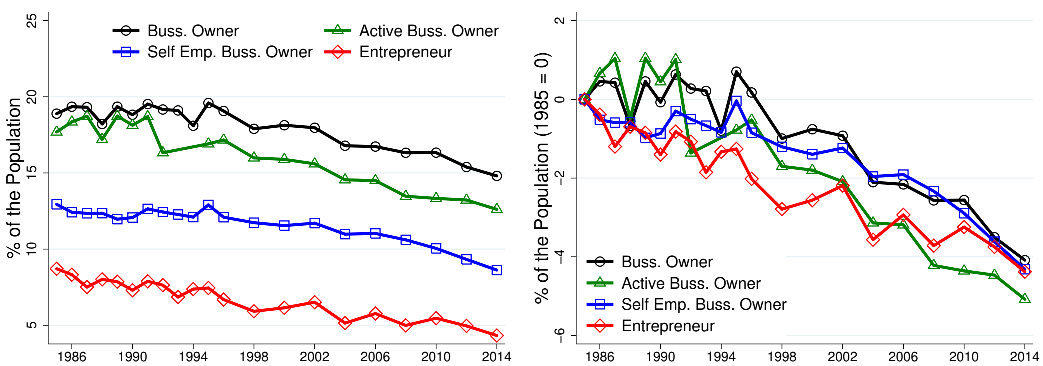
Note: Table A.1 reports statics of a sample of heads of households ages between 22 and 60 years old. See appendix A.1.1 for additional details on the sample selection. Each statistics is the sample average across the waves of 1985 to 2015. Business owners are individuals that declare owning a business. Active business owners declares have a business and have worked for it in a given year. Are active business owners that declare be self-employed. Entrepreneurs are the subset of the previous group that declare to have a managerial or professional occupation. All monetary values are deflated by the PCE index and expressed in 2012 US dollars. Household wealth is defined as the sum if savings checking accounts, bonds, stocks, IRA, housing equity, other real state, and vehicles, minus total debt. All statistics, with the exception of the number of observation, were calculated using sample weights.

Figure A.3: Average of Recent Labor Income



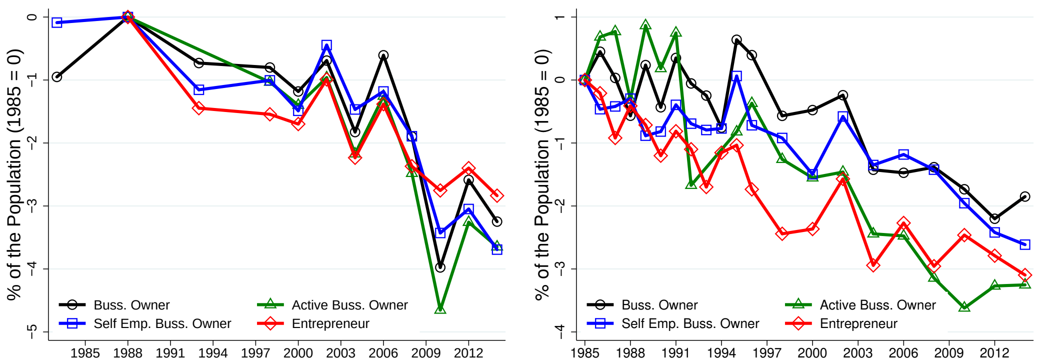
Note: Figure A.3 shows the average (log of) recent labor earnings for a sample of men, heads of household, who are neither a business owner nor self-employed in year t. Recent earnings are defined as the average of the real labor income in periods t, $t-1$, and $t-2$ for years prior 1997 and the average labor income in periods t and $t-2$ after 1997. The left panel shows the average recent earnings within the group of households that become *business owners* in year $t+2$ while the right panel shows the same statistic for individuals that remain as workers in period $t+2$. The difference in the slope in the left and right panels is statistically significant at 1%.

Figure A.4: Proportion of Entrepreneurs



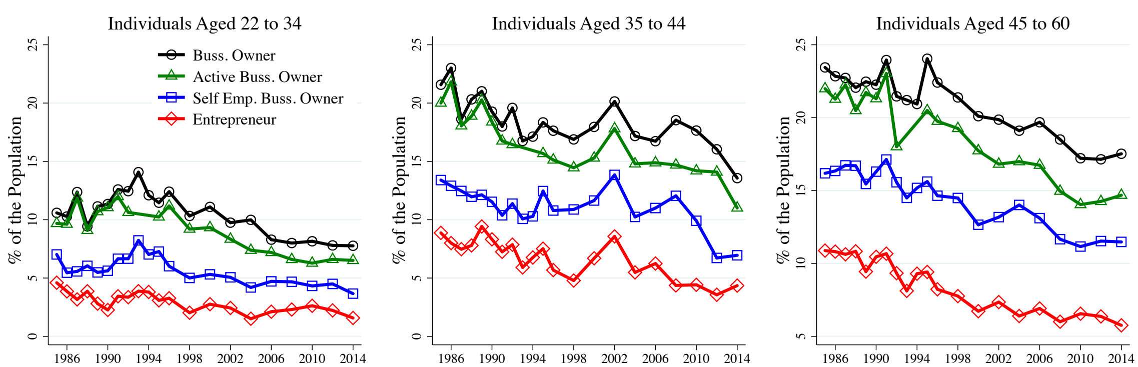
Note: Figure A.4 shows the share of entrepreneurs for different definitions calculated over a sample of employed heads of households. See the note in table 1.1 for more details on the classification of entrepreneurial households.

Figure A.5: Proportion of Entrepreneurs – Additional Definitions



Note: Figure A.5 shows the proportion of households that are neither business owners nor self-employed in period t that are classified as entrepreneurs in period $t+2$ for different definitions of entrepreneurship. The right panel shows the same statistics rescaled to the corresponding value in 1985.

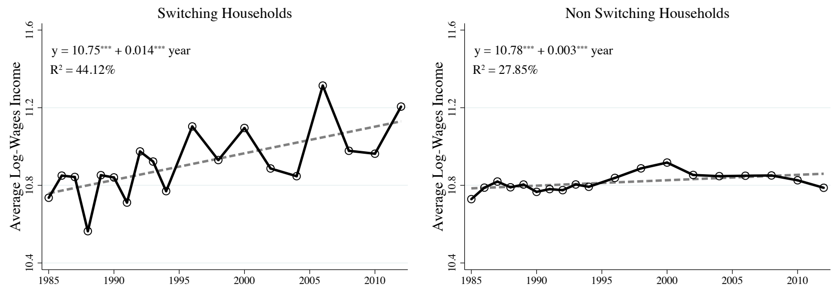
Figure A.6: Share of Entrepreneurs within Different Age groups



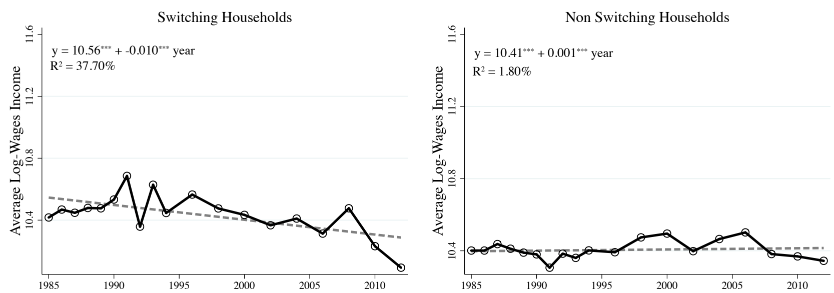
Note: Figure A.6 shows the fraction of entrepreneurs within three different age groups. See notes in table 1.1 for additional details.

Figure A.7: Average Wages and Salaries Income for Workers

(a) Some College or More



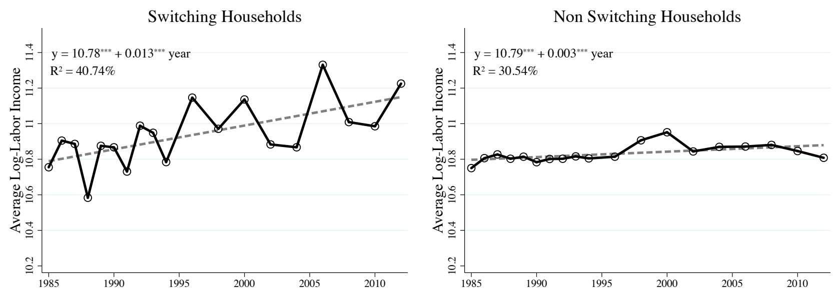
(b) High School Graduates or Less



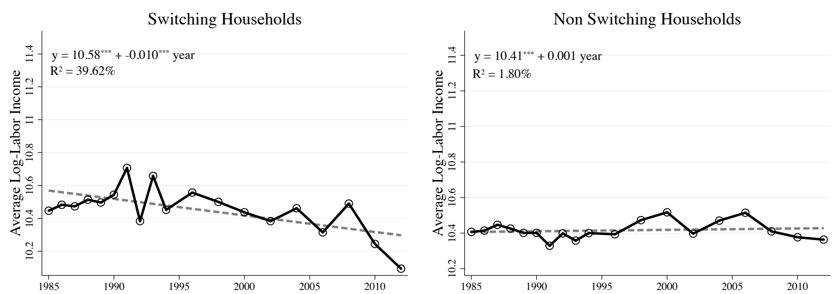
Note: Figure A.7 shows the average log of wages and salaries income of men head household who are neither a business owner nor self-employed in year t from PSID. Top panels show the statistics for college graduates. Bottom panel shows the statistics for workers with some college or less. The left panel shows the average wage within the group of households that become *self-employed business owners* in year $t+2$ while the right panel shows the same statistic for individuals that remain as workers in period $t+2$.

Figure A.8: Average Total Labor Earnings for Workers

(a) Some College or More

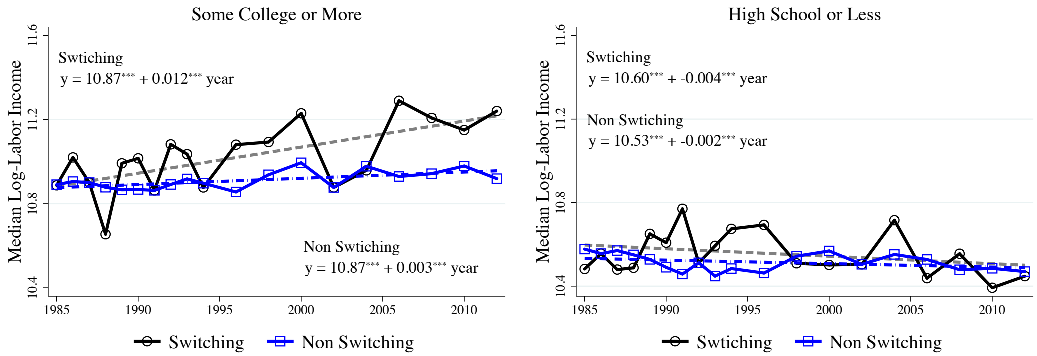


(b) High School Graduates or Less



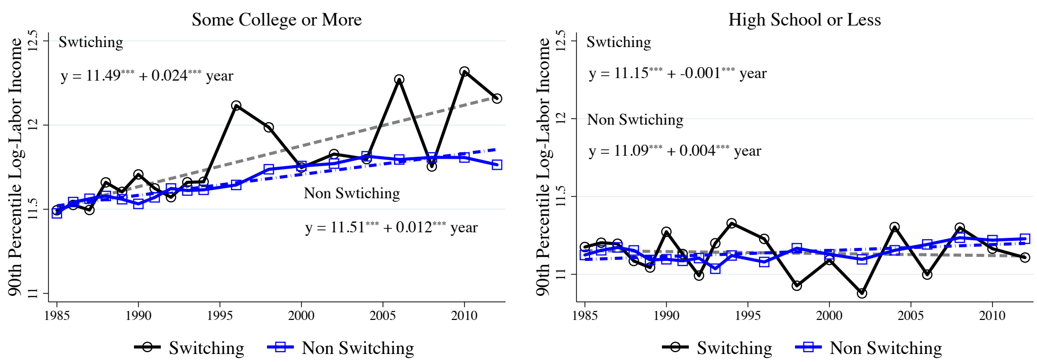
Note: Figure A.8 shows the average of log labor earnings of men, head household who are neither a business owner nor self-employed in year t from PSID. Recent earnings are defined as the average labor income in periods t, $t-1$, and $t-2$ for years prior 1997 and the average labor income in periods t and $t-2$ after 1997. Top panels show the statistics for college graduates. Bottom panel shows the statistics for workers with some college or less. The left panel shows the average wage within the group of households that become *self-employed business owners* in year $t+2$ while the right panel shows the same statistic for individuals that remain as workers in period $t+2$.

Figure A.9: 50th Percentile of the Labor Income For Workers



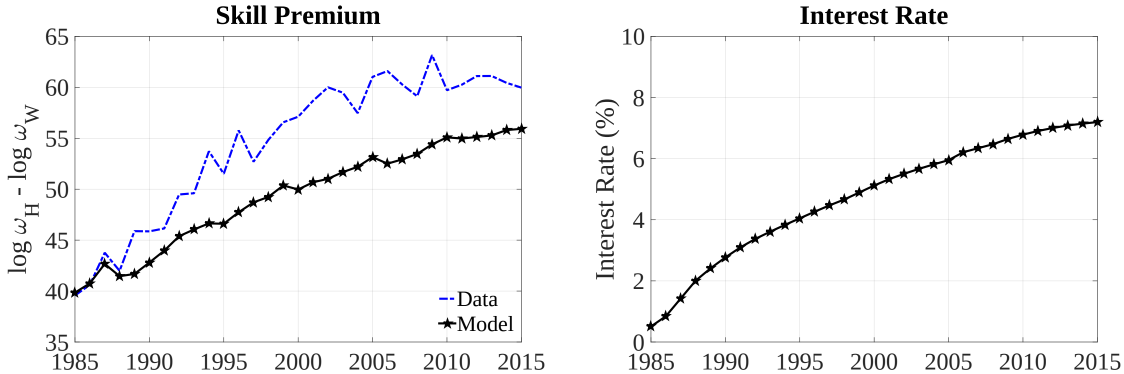
Note: Figure A.9

Figure A.10: 90th Percentile of the Labor Income For Workers



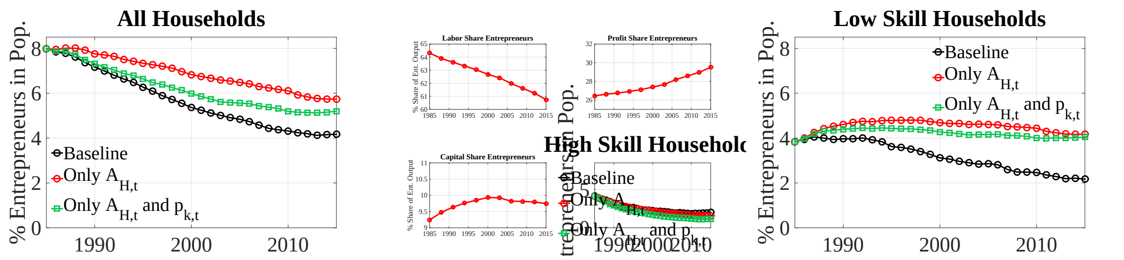
Note: Figure A.10

Figure A.11: College Premium and Interest Rate in GE



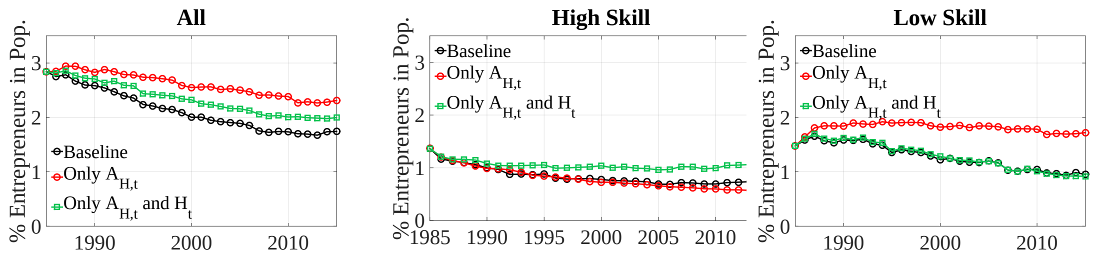
Note: Figure A.11 shows the evolution of the skill premium and interest rate implied by the mode in the general equilibrium case.

Figure A.12: Alternative Decomposition of the Decline in Entrepreneurship



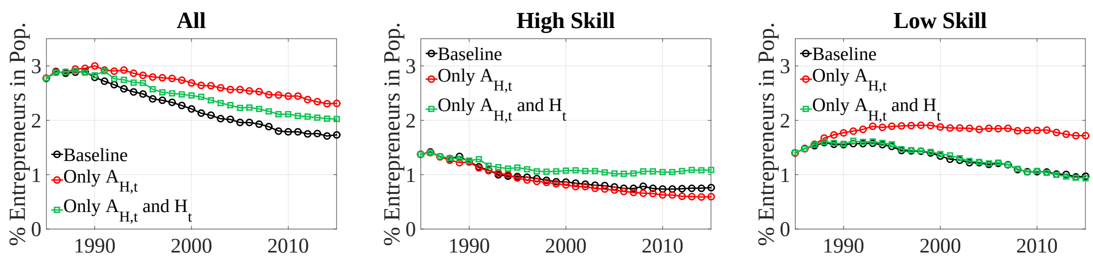
Note: Figure A.12 shows the share of entrepreneurs implied by the model. The black starred line shows the share of entrepreneurs for the baseline case, the red circled line considers only the evolution of $A_{H,t}$, while the green squared line considers $A_{H,t}$ and $p_{k,t}$. The center and right panel show similar statistics for high and low skill workers.

Figure A.13: Decomposition of the Transition Rate



Note: Figure A.13 shows the time series of the transition rate into entrepreneurship implied by the model. The black starred line shows the baseline case, the red circled line considers only the evolution of $A_{H,t}$, while the green squared line considers $A_{H,t}$ and H_t. The center and right panel show similar statistics for high and low skill workers.

Figure A.14: Decomposition of the Exit Rate



Note: Figure A.14 shows the time series of the transition rate out from entrepreneurship implied by the model. The black starred line shows the baseline case, the red circled line considers only the evolution of $A_{H,t}$, while the green squared line considers $A_{H,t}$ and H_t. The center and right panel show similar statistics for high and low skill workers.

A.5 SFC Sample

In this section, I describe in more detail the sample selection the variable construction for the results using the Survey of Consumer Finances (SCF). The SCF is a nationally representative survey conducted every three years by the Federal Reserve Board of Governors. Importantly, the SCF oversample rich individuals which are more likely to be entrepreneurs. I take data from over the period 1989 to 2016. The raw sample contains 238,880 individual-year observations. For comparability to the results on the PSID, I consider a sample of heads of households between 22 and 60 which are in the labor force with valid information on education. Following Cagetti and De Nardi (2006) and Michelacci and Schivardi (2016)I classify an individual as an entrepreneur if she is self-employed in his primary job (variable X4106 in the SCF) and she has an active management role in at least one privately owned business (variable X3104 in the SCF). I further divide the sample on individuals with a high school degree or less and those with some college studies or more (variable X5901 in the SCF for the period 1989 to 2013 and X5931 for 2016). Finally, I drop all those individuals that do not work for a pay (variable X4106 in SCF). This leave us with a sample of 173,066 individual-year observations. For all the calculations I use the sample weights (variable X42001).

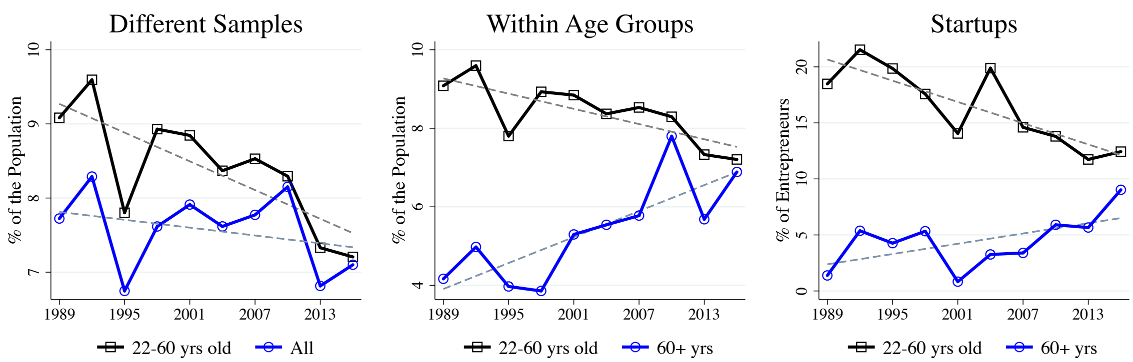
The main body of the text presented the share of entrepreneurs in the age group of 22 and 60 years old and within education groups (1.6). Here I discuss two additional issues. First, Michelacci and Schivardi (2016) use the SCF to study the returns to entrepreneurs and he suggests that the share of entrepreneurs has been stable from 1989 to 2013. The difference between my conclusions and theirs steams from the sample selection. Given their focus on entrepreneurial returns Michelacci and Schivardi (2016) consider the entire population sample while I focus on 22 to 60 years old individuals which are more likely to switch occupations between workers and entrepreneurs. The left panel of figure A.15 shows the share of entrepreneurs in the SCF with and without the group of individuals that are over 60 years old. In the second case, the share of entrepreneurs is more or less stable over the sample period. This is because within the group of over 60 years old, the share of entrepreneurs is either flat or increasing as it is shown in the center panel of figure A.15. This, coupled with an increasing share of this group in the US population, pushes the share of entrepreneurs up, keep the share more or less constant. In contrast to Michelacci and Schivardi (2016), I focus on individuals that might transit into entrepreneurship and weigh the costs and benefits of starting a new firm. Importantly, for the group of entrepreneurs that are between 22 and 60 years old, the share of startups entrepreneurs, that is, the fraction of entrepreneurs whose primary firm is one year old or less, shows an step decline during the sample period while this share is smaller and increasing within the group of entrepreneurs older than 60 years (right panel of figure A.15).

Table A.2: Distribution of Entrepreneurs Across Industries

Industry	Less than College	College Graduate	All
Agro, Fish, and Forestry	14.70	3.1	7.3
Construction	25.04	7.39	13.53
Manufacturing	7.71	6.86	7.16
Retails and Wholesale Trade	17.26	11.53	13.53
Professional Services	14.39	25.54	21.66
Transportation, Comm. and Utilities	19.98	43.74	35.47
Others	0.92	1.85	1.53

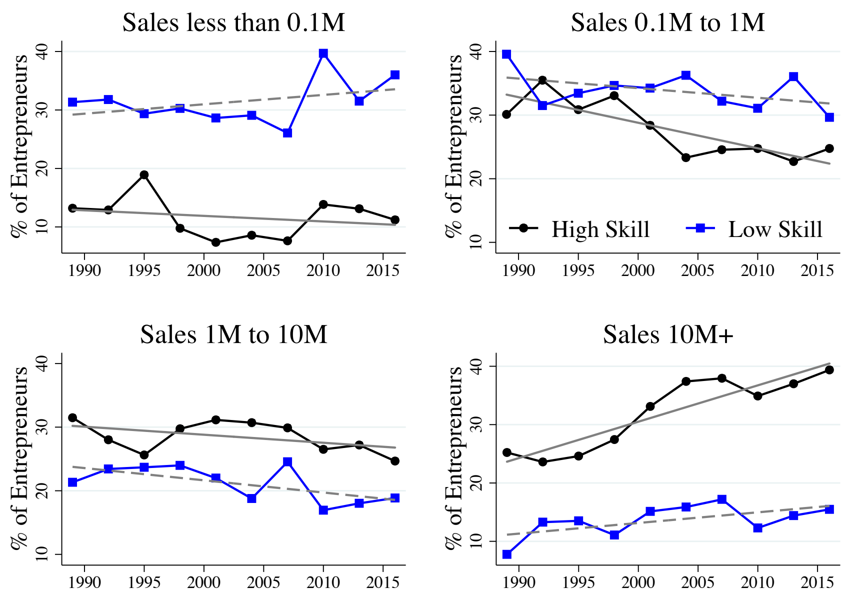
Note: table A.2 shows the distribution of entrepreneurs across 1-digit SIC sectors in the SCF. Entrepreneurs are defined as heads of household that own a businesses and declare to have an active management role in the business. SIC sectors are defined using the first main business owner by the household.

Figure A.15: Share of Entrepreneurs and Startups in the SCF



Note: Figure A.15 shows the share of entrepreneurs within different population groups and for different definitions of entrepreneurs. The left panel shows the share of entrepreneurs in the baseline sample (individuals between 22 and 60 years old) and considering the entire sample (individuals of 22 years or more). The center panel shows the share of entrepreneurs within age groups. The right panel shows the share of startups entrepreneurs within age groups. Startup entrepreneurs are entrepreneurs that actively manage at most two firms and one of them has at most 1 year old.

Figure A.16: Size Distribution: High and Low Skill Entrepreneurs



Note: The figure A.16 shows the evolution of the share of entrepreneurs for different size classifications within high and low skill entrepreneurs. All monetary values were deflated using the PCE and expressed in 2012 dollars.

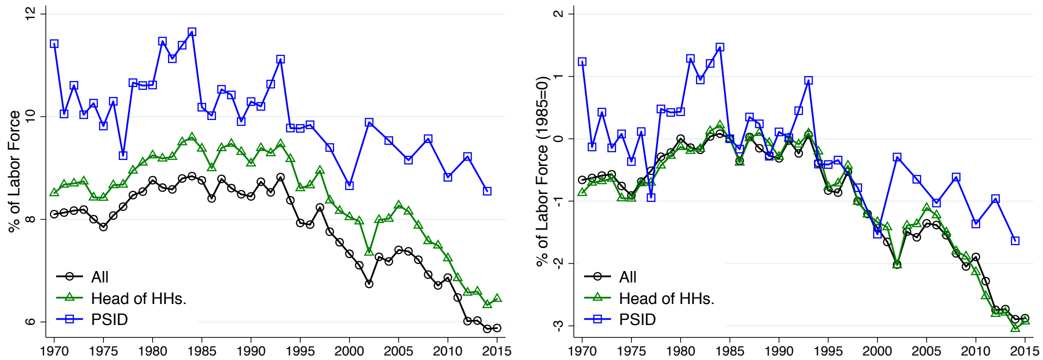
A.6 Evidence from CPS

The decline in the share of entrepreneurs documented in section 1.2 comes from a small, although nationally representative, sample of household. Hence, one might wonder whether the results presented here using the PSID can also be observed in other data sets. For this reason, in this appendix a draw a sample of household from the CPS from 1970 to 2015. The CPS is a nationally representative survey collected by the US Census Bureau. Here, I use the March supplement that collects information on employment status, income, industry, and occupation, to analyze if the patters found in the PSID are also present using a much larger data set. As much as possible, I keep the same sample selection used in the previous section. The main drawback of using the CPS is that the definition of what constitutes an entrepreneur can be based only on few questions that mostly refer to whether or not the individual is self-employed, and therefore, the sample could be skewed to individuals that work for themselves and do not hire any other employees. This is important for two reasons. First, most of the evidence presented by Haltiwanger et al. (2015), Decker et al. (2016), and others refer to employee firms and therefore self-employed individuals that do not hire other workers are not considered. Secondly, new empirical evidence suggests that alternative works agreements (contractors, part time workers, etc.) are in a rise in the US economy (Katz and Krueger (2016)). To the extent that there is overlap between self-employed individuals and workers in alternative work agreements overlap, analysis trends of the share of self-employed might be misleading. With these caveats in mind, I consider two measures of entrepreneurship. The first is the proportion of individuals that are self-employed over the entire population, and second, to have a closer definition to the one used in PSID, I consider the fraction of self-employed head of households.

The left panel of figure A.17 shows that the share of self-employed in the population has steadily declined since the early 1980s and such decline has accelerated since the mid 1990s. For better comparison with my previous results, here I also show the proportion of self-employed head of households from the PSID. The levels are somewhat different, with a larger proportion of self-employed in the PSID, but the decline is similar in both data set, as it is shown in the right panel of A.17. Figures A.18 and A.19 complement these results showing the decline in the share of self-employed within education and age groups.

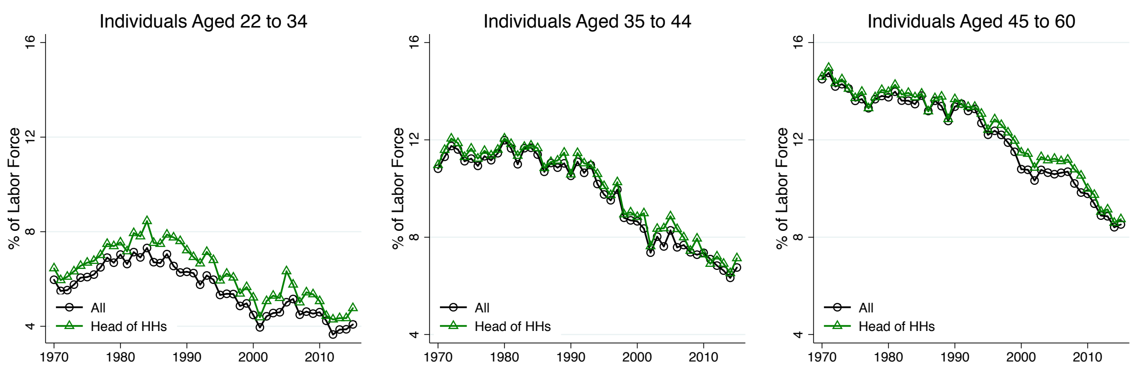
Because the CPS is a much larger sample, we can go one step further and study in which which sectors the decline in the share of self-employed is more evident. For doing that, I calculate the share of self-employed individuals within 14 different sectors. The employment share accounted for self-employed is quite different across industries, as one can expect from the large disparities in the scale of production. For instance, the share of total employment account for by self-employed workers in services is around 15%, while in manufacturing it is 1.5%. To have a better comparison across sectors, figure A.20 shows the share of self-employed workers within each industry rescaled to its value in 1985. With the exception of manufacturing, the decline in the share of self-employed is quite evident in almost all sectors. Interestingly, the decline in self employment is not circumscribed to sectors such as retail and whole sale trade (see upper right panel) which has been increasingly dominated by big retail stores, but it is also present in construction or even within growing sectors , such as Services (see the upper left panel).

Figure A.17: Share of Self-employed in the Population



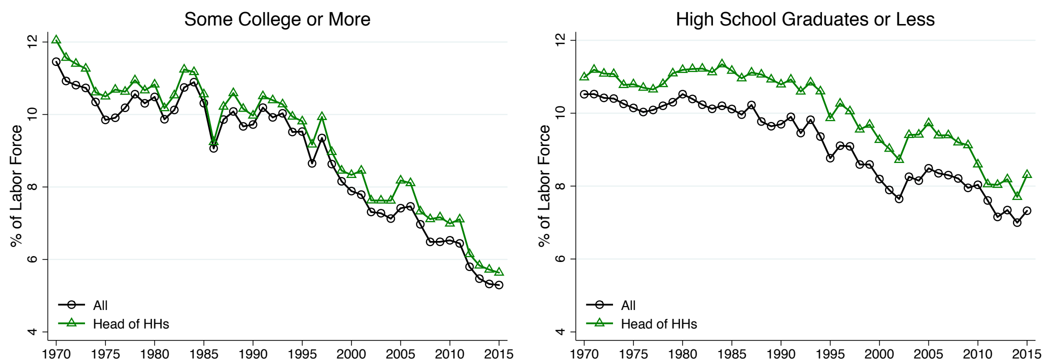
Note: Figure A.17 shows the proportion of self-employed individuals aged between 22 and 60 years old. Individuals which are not in the labor force (students, disable) or are in the military are excluded. The share of self-employed head of households is the ratio to all the head of households that are self-employed over the population of head of households.

Figure A.18: Proportion of Self-employed by Age – CPS



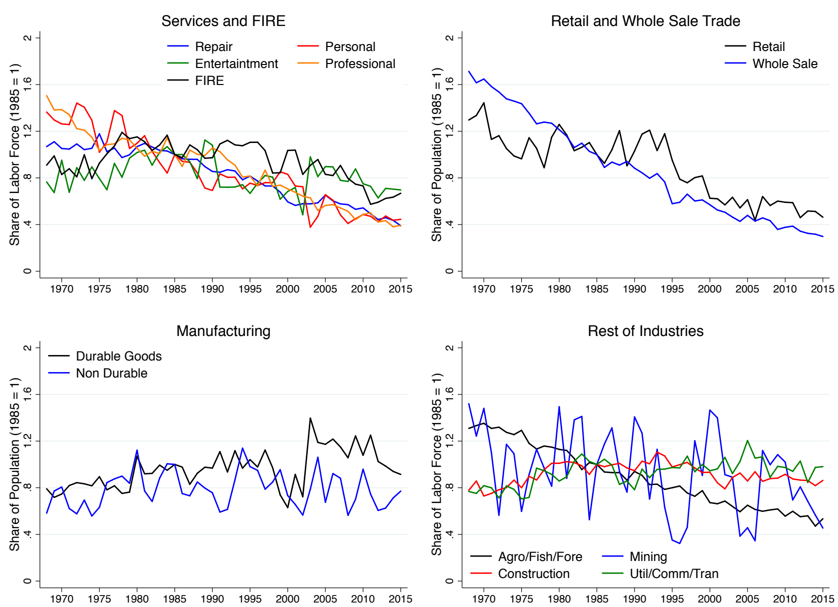
Note: Figure A.18 shows the proportion of self-employed individuals within age groups. Individuals which are not in the labor force (students, disable) or are in the military are excluded. The share of self-employed head of households is the ratio to all the head of households that are self-employed over the population of head of households within each age group.

Figure A.19: Proportion of Entrepreneurs by Education – CPS data



Note: Figure A.19 shows the proportion of self-employed individuals within education groups. Individuals which are not in the labor force (students, disable) or are in the military are excluded. The share of self-employed head of households is the ratio to all the head of households that are self-employed over the population of head of households within each education group.

Figure A.20: Proportion of Self-Employed by Industry – CPS data



Note: Figure A.20 shows the proportion of self-employed individuals within industry sectors. Individuals which are not in the labor force (students, disable) or are in the military are excluded. The share of self-employed head of households is the ratio to all the head of households that are self-employed over the population of head of households within each industry.

Appendix B

Appendix to Chapter 2

B.1 Appendix: Data Sources and Variable Construction

This appendix describes the data sources and sample selection. Firm-level data for the United States comes the Census Bureau's Longitudinal Business Statistics (LBD) and Compustat. For the cross-country comparisons, we use firm-level data available in the Bureau van Dijk's Osiris database and Global Compustat. The online appendix and replication packet–available on the author's websites–contains further details of the construction of the sample and moments calculation.

B.1.1 United States: Longitudinal Business Database

We construct measures of employment growth at the firm-level using the Census Bureau's Longitudinal Business Database (LBD). The LBD covers the universe of establishment in the nonfarm private sector in the United States from 1976 to 2015. It provides detailed establishment and firm-level information on employment, payroll, location, firm age, industry, legal form of organization, etc.. Crucially, firm and establishment identifiers in the LBD allow us to construct measures of employment growth at different time horizons. From the LBD, we select a sample of establishments that, in a given year, have nonnegative, non-missing employment and payroll and have valid industry data. We then sum up the employment within the same firm to construct an annual measure of employment. We measure the growth rate of employment of firm j in period t as the log-difference between periods t and $t+k$, $g^e_{j,t} = \log E_{j,t+k} - \log E_{j,t}$ where $k \in \{1,3,5\}$ and by the arc-percent change between the same periods.

Calculating the Kelley skewness requires the computation of specific different percentiles of the distribution sales growth distribution. Notice that a percentile provides information of a particular firm, which violates the disclosure criteria imposed by the Census Bureau. Hence, to avoid the disclosure of any sensitive information, we calculate the *pth* percentile of the employment growth distribution as the employment-weighted average on a band of +1 and -1 percent centered around *pth*. For instance, the 90th percentile of the distribution is the weighted average of the employment growth across all observations between the 89th and 91st percentiles of the distribution, both ends included. We proceed in the same way to construct the 10th and 50th percentiles of the distribution and use these values to calculate the Kelley skewness. All measures are weighted by the average employment of the firm between periods t and $t+k$, that is $\overline{E}_{j,t} = 0.5 \times (E_{j,t+k} + E_{i,t})$. The massive sample size of the LBD ensures that the sample used to calculate each of the percentiles is large enough to have an accurate approximation to the actual quantiles of the distribution.

We also use the LBD to compare the distribution of employment growth between recessions and expansions years using kernel density estimation. The sample selection is the same used in the rest of our results, however, the Census Bureau requires to drop the bottom and top 5% of the distribution. The kernel densities presented in figure 2.3 were calculated over the remaining sample.

B.1.2 United States: Compustat

For the United States, we construct time-series of the cross-sectional dispersion and skewness of the sales growth distribution and the distribution of stock returns. To construct the time-series of the sales growth distribution we proceed as follows. We begin by retrieving firm-level data of net sales, and other variables at a quarterly frequency, and employment at an annual frequency, from Compustat from 1964q1 to 2017q4 available at WRDS database. The raw dataset of sales (Compustat variable saleq) and stock prices (Compustat variable prccq) contains more than 1.7 million quarter-firm observations with an average of approximately 4,660 firms per quarter. We drop all observations with negative sales, repeated observations, and incorporated outside the United States (we keep observation with Compustat variable fic equal to "USA"). We also drop all observations that do not have a SIC classification or with a classification above 90. Then, we deflate nominal sales by the CPI (FRED series CPIAUCSL), and we calculate the growth rate of sales as the log difference and the arc percentage change between quarter t and $t+k$ with $k \in \{4, 12, 20\}$. This leaves us with around 1 million sales growth (log difference) observations. For our main results, we consider firms with at least 10 years of data on quarterly sales (40 quarters, not necessarily continuous), which further reduces the sample to 819,977 observations between 1970q4 and 2017q2, with an average of 5,359 firms per quarter. Finally, in each quarter we calculate different cross-sectional moments discussed in the main body of this document. Our main sample considers firms with at 10 years of data (40 quarter) although our results remain robust if we drop this restriction or if we consider firms with 25 years of data. When accounting for entry and exit of firms using the arc-percentage change, for each firm we add an observation upon entry (equal to 2) and one additional observation upon exit (equal -2) under the assumption that before entering and after exit, the firm has a value of sales or employment equal to 0. We consider entry firms as newly listed firms while exiting firms are those delisted in a particular period, independent of the reason (M&A, bankruptcy, or any other).

For our result at the annual frequency, we follow the same sample selection. The raw annual dataset contains 500,004 year/firm observations. We drop all observations with negative sales and duplicated entries, with missing SIC classification or two digit SIC above 90. We deflate nominal variables using CPI (FRED series CPIAUCSL) and we calculate the growth rate of sales (Compustat variable sale) and employment (Compustat variable emp) as the log change between year t and $t+k$ with $k \in \{1, 3, 5\}$. This leaves us with 266,192 firm/year observation (sales growth) between 1970 and 2016, with an average of 5,663 firms per year. Our main sample consider only firms with at the least 10 years data (not necessarily continuous) but our results remain robust if we drop this restriction or if we consider firms with at least 25 years of data. When accounting for entry and exit of firms using the arc-percentage change, for each firm we add an observation upon entry (equal to 2) and one additional observation upon exit (equal -2) under the assumption that before and after exit, the firm has a value of sales or employment equal to 0. We consider entry firms as newly listed firms, while exiting firms are those delisted in a particular period, independent of the reason (M&A, bankruptcy, or any other). We complement this data with macroeconomic series from FRED (real gross domestic product per capita, FRED series A939RX0Q048SBEA).

B.1.3 Cross-Country: BvD Osiris and Global Compustat

Cross country firm-level panel data on sales and employment come from the Bureau van Dijk's Osiris database. Osiris is a database of listed public companies, commodity producing firms, banks, and insurance companies from over 190 countries. The combined industrial company dataset contains financial information for up to 20 years and 80,000 companies. In our analysis, we focus on the industrial dataset.

The raw dataset contains 977,412 country/firm/year observations from 1982 to 2018. We then drop all observations with missing or negative sales, we clean all duplicated entries, and firms with missing NAIC classification. We transform all observations into US dollars using the exchange rate reported in the same database. Then, we deflate nominal sales using US annual CPI and calculate the growth rate of real sales as the log change and arc percentage change between years t and $t + k$ with $k \in \{1, 3\}$. This leaves us with 748,574 observations (log change of sales). We further restrict the sample to firms with more than 10 years of data; country/year cells with more than 100 observations; countries with more than 10 years of data; and years with more than 5 countries. This sample selection reduces the dataset to an unbalanced panel of 678,563 observations in 45 countries between 1989 and 2015. We complement this data with real GDP in US dollars from the World Bank's World Development Indicators database.

The data on daily stock prices come from the Global Compustat database, which provides standardized information on publicly traded firms for several countries at annual, quarterly, and daily frequencies. The raw data contain firm-level observations of daily stock prices between 1985 and 2018 for 48 countries. We drop all duplicated observations and drop all firms with less than 2000 observations (firms with approximately 10 years of data). Then we calculate daily price returns as the log difference of the stock price between two consecutive trading days. We apply a similar sample selection, keeping firms with at least 10 years of daily price data. The total sample contains an unbalanced panel of 44 countries from 1985 to 2017 from which we drop all country quarter with less than 100 firms. The final data contains a total of 29 countries from 1985 to 2017. Then, within each quarter, we calculate the cross-sectional moments of the daily stock price distribution. We complement this dataset with per capita GDP growth form World Bank's World Developing Indicators and quarterly GDP growth from the OECD Stats.

B.2 Appendix: Additional Robustness Results

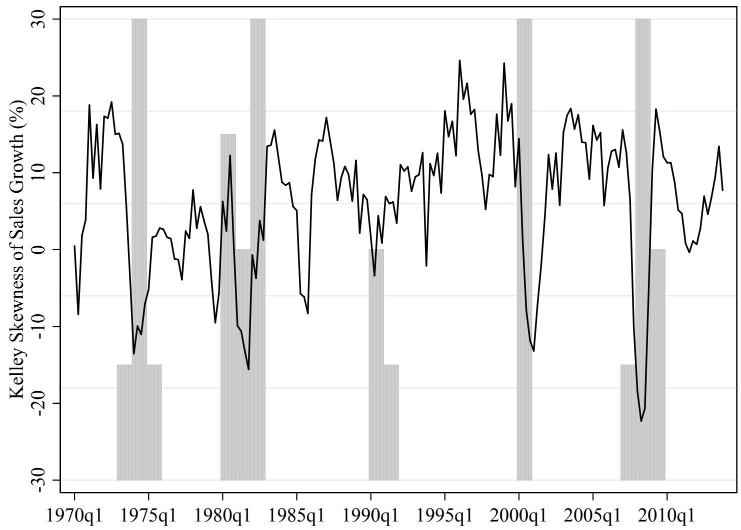
Table B.1: Skewness is Lower During Industry Recessions

Kelley Skewness of Three-Years Growth Rate of Firms' Outcomes						
Sample:	United States			Cross-Industry		
	(1)	(2)	(3)	(7)	(8)	(9)
Source	CSTAT	LBD	CSTAT	CSTAT	CSTAT	CSTAT
	Sales	Emp.	Stock Price	Sales	Emp.	Stock Price
$\Delta GDP_{i,t}$	2.89^{**}	1.14^{**}	3.96^{***}			
	(1.32)	(0.50)	(1.12)			
Industry Growth				5.92^{***}	6.02^{***}	0.70
				(1.60)	(1.67)	(1.17)
R^2	0.01	0.15	0.10	0.12	0.11	0.09
N	182	35	180	3,652	930	3,602
Freq.	Qtr	Yr	Qtr	Qtr/	Yr	Qtr
F.E.	N	N	N	Qtr/Ind	Yr/Ind	Qtr/Ind
Sample	640K	-	650K	780K	193K	651K

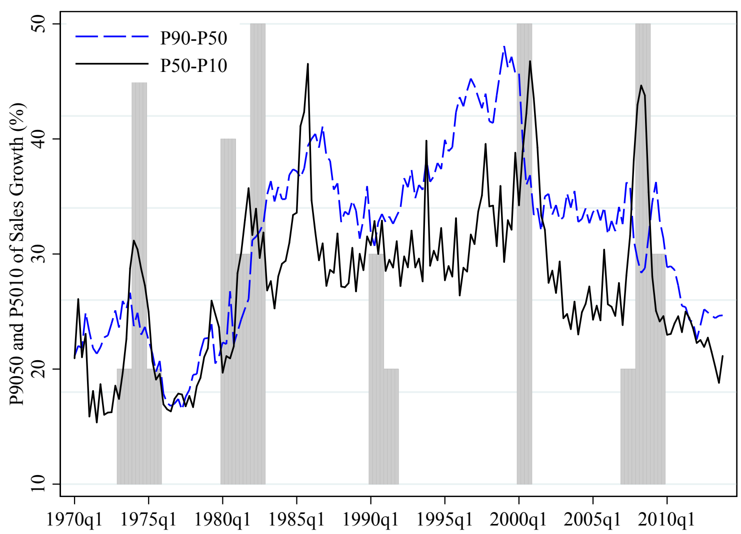
Note: Table B.1 shows a series of industry-level panel regressions. In each column, the dependent variable is the cross sectional Kelley skewness of the growth rates of real quarterly sales, annual employment growth, and quarterly stock returns distribution within period-industry cells defined by 2-digit NAICS (total of 22 industries) for a sample of publicly traded firms from the Compustat dataset. The independent variable, $\overline{\Delta S}_{j,t}$, is the average of the sales growth distribution within the period-industry cell. LBD moments were calculated weighting by firm-size. In all regressions, the sample period is 1970 to 2017 and consider a full set of period and industry fixed effects. Row labeled Sample corresponds to the total firm-period observations used to calculate the cross sectional moments. N corresponds to the number of period-industry observations used in the regressions. Standard errors in parentheses below the point estimates are clustered at the NAIC-2 industry level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Figure B.1: The Skewness of firm-level Quarterly Sales Growth is Procyclical

(a) Compustat: Skewness of Sales Growth Distribution



(b) Compustat: Upper and Lower Tail Dispersion of Sales Growth



Note: The top panel of figure B.1 shows the time-series of the cross-sectional Kelley skewness of the distribution of firm employment growth for a sample of firms from LBD. Moments are weighted by the average firm employment size between years t and $t+1$. The bottom panel of figure B.1 shows the time-series of the cross-sectional Kelley skewness of the distribution of firm quarterly sales growth for a sample of publicly traded firms from Compustat. The shaded bars represent NBER recession periods. See Appendix B.1.1 for additional details on the sample construction and moment calculations in the LBD and Compustat.

Table B.2: Dispersion of Firm's Outcomes is Higher During Recessions

	90th-to-10th Percentiles Spread of the Growth Rate of Firm's Outcomes							
	United States					Cross-Country		
	(1)	(2)	(3)	(4)	(5)	(7)	(8)	(9)
	Firm Sales		Stock Returns		Firm Emp.	Firm Sales	Firm Stock	Firm Emp.
	One Year	Three Year	One Year	Three Year	One Year	Growth	Returns	Growth
$\Delta GDP_{i,t}$	-3.91***	2.55**	-3.93**	-4.78***	0.93*	-0.79	-1.84	-0.76
	(1.14)	(0.99)	(1.62)	(1.78)	(0.50)	(0.59)	(1.79)	(0.74)
N	184	182	180	180	39	838	4,306	824
Freq.	Qtr	Qtr	Qtr	Qtr	Yr	Yr	Qtr	Yr
F.E.	N	N	N	N	N	Yr/Ctry	Qtr/Ctry	Yr/Ctry
Source	CSTAT	CSTAT	CSTAT	CSTAT	LBD	BvD	GCSTAT	BvB

Note: The left panel of table B.2 shows a series of time-series regressions in which the dependent variable are the 90th-to-10th percentiles spread of the one-year and three-year growth rate of sales growth (columns 1 and 2), stock returns (columns 3 and 4), and employment growth (columns 5 and 6) for a sample of firms from Compustat (columns 1 to 4) and the LBD (columns 5 and 6). Compustat data covers the period 1970 to 2017 whereas LBD data covers the period 1976 to 2015. In each regression, the independent variable is the annual growth rate of quarterly GDP per capita. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. The right panel of table B.2 shows a series of country-panel regressions where the dependent variable is the within-country P90-P10 spread of firm-level sales growth, stock returns, or employment growth. The independent variable is the growth rate of GDP per capita at the country level. Sales and employment data is obtained from the BvD Osiris database whereas stocks returns are obtained from Global Compustat. All cross-sectional moments where calculated weighting growth rate observations by firm size. All regressions consider a full set of time and country fixed effects. The raw labeled Sample shows the underlying sample of firms used to calculate the cross-sectional moments. LBD sample size is not disclosed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

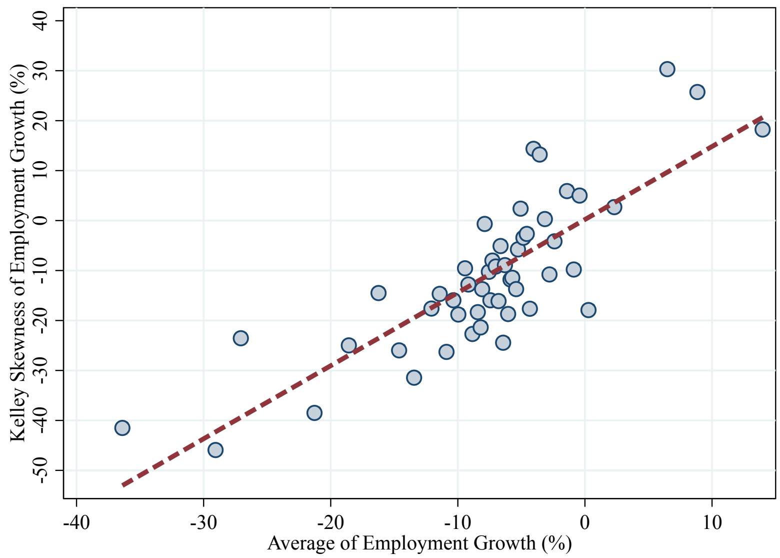
Table B.3: Higher Order Moments of Firm's Outcomes

	Kelley Skewness					Crow-Siddiqui Kurtosis			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Residual Sales		Sales per Employee		Sales Deviation	Firm Sales		Stock Returns	
	One Year	Three Years	One Year	Three Years		One Year	Three Years	One Year	Three Years
$\Delta GDP_{i,t}$	3.80***	1.480	3.01**	4.17***	1.46**	0.36***	−0.19	0.16***	−0.21*
	(1.42)	(0.91)	(1.23)	(0.97)	(0.58)	(0.08)	(0.12)	(0.06)	(0.13)
N	178	178	174	166	47	184	182	180	180
Freq.	Qtr	Qtr	Qtr	Qtr	Yr	Qtr	Qtr	Qtr	Qtr
Sample	500K	500K	500K	500K	113K	640K	640K	650K	650K
Source	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT

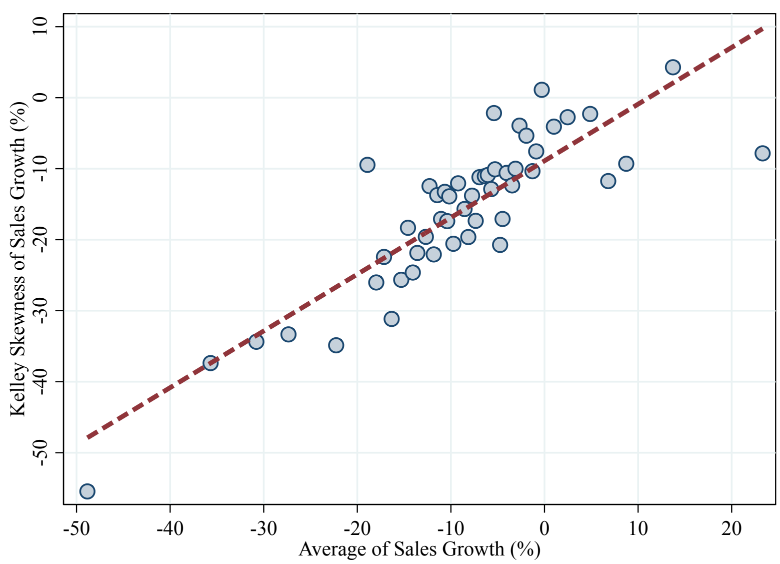
Note: The left panel of table B.3 shows a series of time-series regressions for the United States in which the dependent variable is the Kelley Skewness of the one-year and three-years growth rate of residualized sales growth (columns 1 and 2) and the growth rate of sales-per-employee (columns 3 and 4) for a sample of firms from Compustat. In columns (1) and (2) we have orthogonalized the growth rates of sales from time fixed-effects, firm-fixed effect, size, and other firm-level observable characteristics. Column (5) shows the correlation of GDP growth and the cross-sectional skewness of the deviation of annual firms' sales from a HP trend. Compustat data covers the period 1970 to 2017. The dependent variable in columns (6) to (9) is the Crow-Siddiqui measure of Kurtosis defined as $CKU_t = \frac{P97.5_t - P02.5_t}{P75_t - P25_t}$. In each regression, the independent variable is the annual growth rate of quarterly GDP per capita. All firm-level moments were calculated weighting growth rate observations by firm size measured by the average sales of the firm between periods t and $t+k$. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Figure B.2: THE SKEWNESS FIRMS' OUTCOMES IS LOWER DURING COUNTRY CYCLES

(a) Cross-Country: Firm-Level Employment Growth



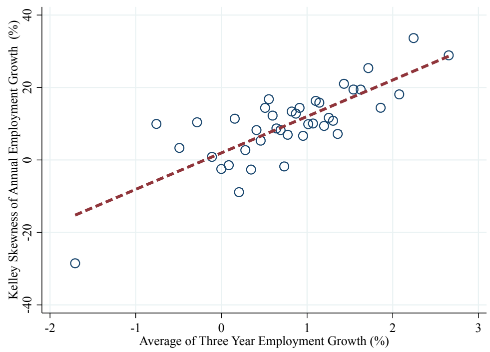
(b) Cross-Country: Firm-Level Sales Growth



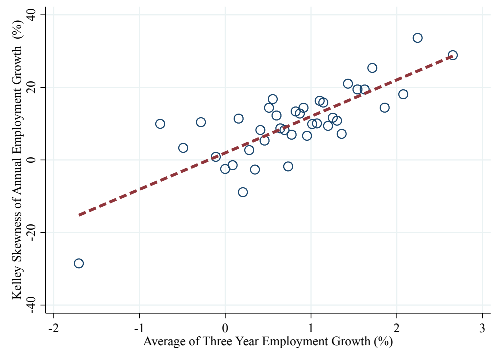
Note: Figure B.2 shows bin scatter plots of the Kelley skewness and average employment and sales growth. The figure is based on an unbalanced panel of firms from the BvD Amadeus database in the following European countries: AUT, BEL, BLR, CHE, DEU, DNK, ESP, FIN, FRA, GBR, GRC, HUN, IRL, ISL, ITA, NLD, NOR, POL, PRT, SWE, UKR. The data covers years 2000 to 2015. BvD Amadeus contains private and publicly traded firms. We apply the same selection criteria we use for the rest of the data.

Figure B.3: The Skewness of Several Firms' Outcomes is Lower During Industry Cycles

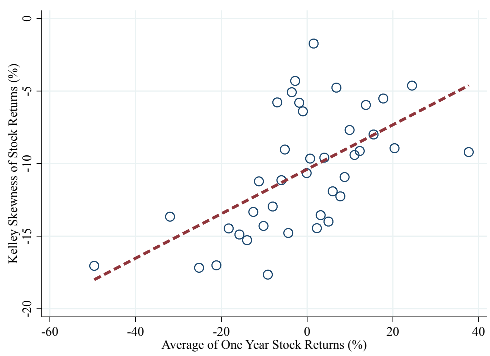
(a) Three-year Sales Growth



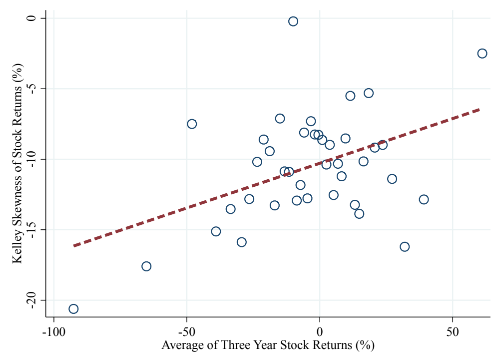
(b) Three-year Employment Growth



(c) One-year Stock Returns



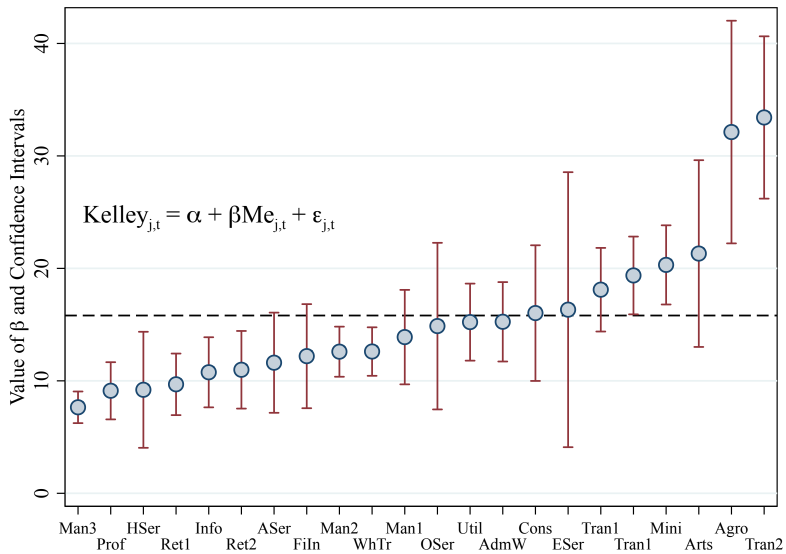
(d) Three-year Stock Returns



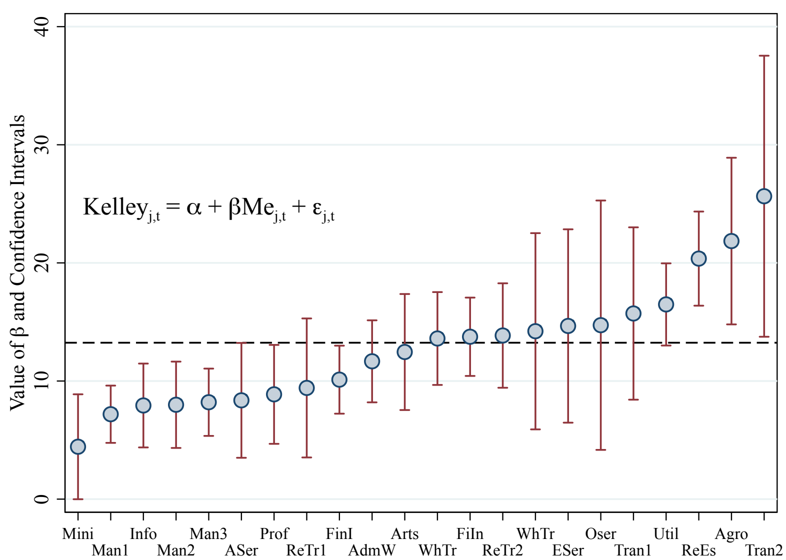
Note: The top left panel of figure B.3 displays a bin scattered plot showing the relation between the within-industry business cycle, measured by the average growth rate of sales, and the within-industry skewness, measured by the Kelley skewness of sales growth for a sample of Compustat firms. Each dot is a quantile of the industry-year distribution of average sales growth. The rest of the plots show similar statistics for employment growth and stock returns.

Figure B.4: The Skewness Firms' Outcomes is Lower During Within-Industry Cycles

(a) Industry: Firm-Level Employment Growth



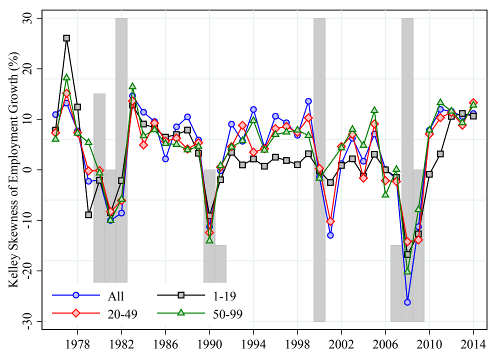
(b) Industry: Firm-Level Sales Growth



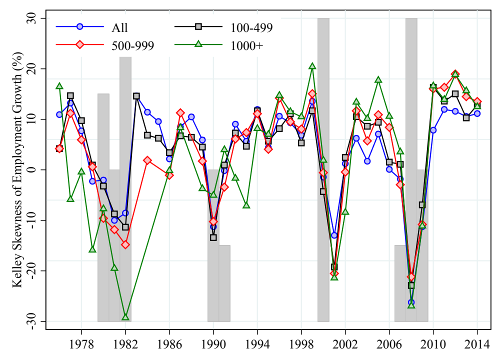
Note: Figure B.4 shows the coefficients and confidence intervals for within-industry regression of the cross-sectional Kelley skewness on the average growth of employment (top panel) and sales (bottom panel) for a sample of publicly traded firms from Compustat. Each industry regression includes a linear trend. Confidence intervals are calculated at 95% of significance. Industries are defined as 2-digit NAIC. In each plot, the dashed line is the coefficient of a panel regression of within industry skewness and average firm growth controlling form time and fixed effect. See Appendix B.1.1 for additional details on the sample construction and moment calculations in Compustat.

Figure B.5: Skewness of Employment Growth Distribution within Establishment Groups

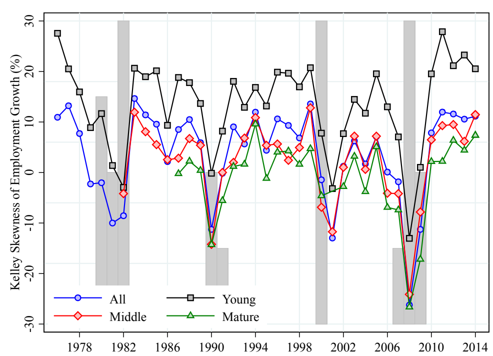
(a) Census LBD: Small Establishments



(b) Census LBD: Medium & Large Establishments



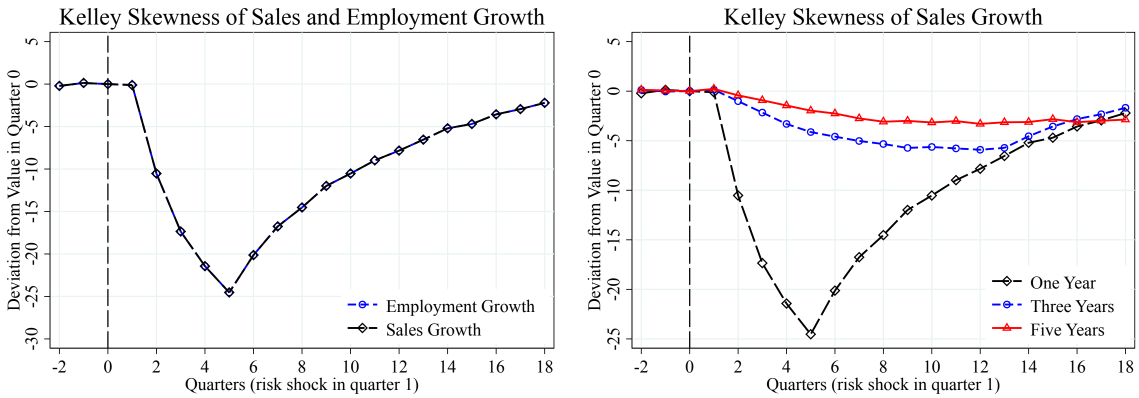
(c) Census LBD: Establishment Age



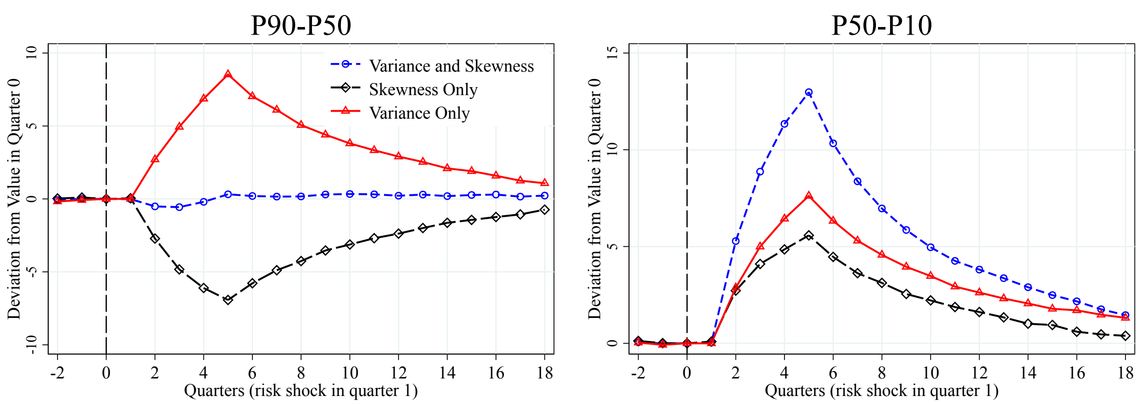
Note: Figure B.5 is based on the Longitudinal Business Database, LBD. The top left and right panels show the skewness of the distribution of establishment-employment growth within different establishment size groups defined by establishment average employment calculated for each establishment i as $\overline{E}_{i,t} = 0.5 \times (E_{i,t} + E_{i,t+1})$; The bottom panel shows the skewness of the distribution of establishment-employment growth within different establishment age groups. Young establishment are those of less than five years, Middle-aged establishment are those between six and ten years, whereas Mature establishment are those of more than ten years old. Establishment already in the sample in 1976 were not considered in any of these groups. All moments weighted by establishment size defined by $\overline{E}_{i,t}$. In all plots, the blue line with circles is the skewness of employment growth for all establishment in the sample.

Figure B.6: Model Generated Moments

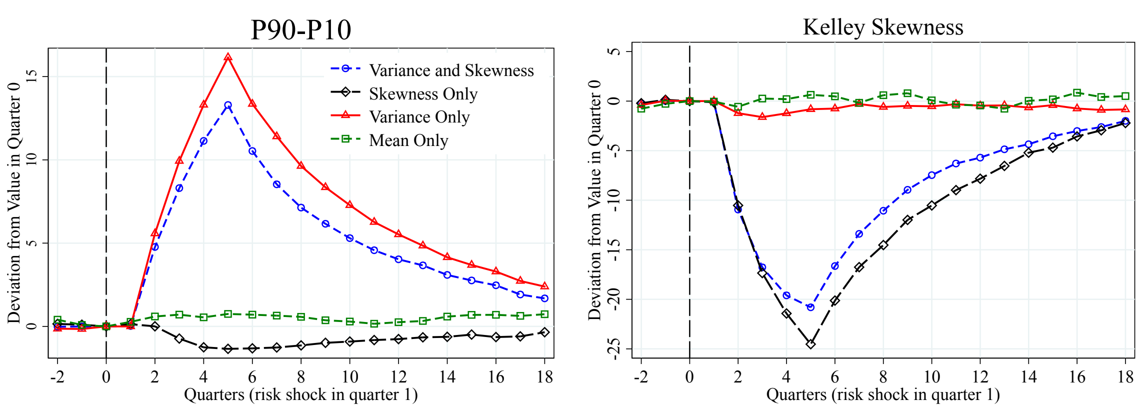
(a) Skewness of Employment and Sales Growth



(b) Right and Left Tail Dispersion of Sales Growth



(c) Aggregate Productivity Shock does not Affect Dispersion or Skewness of Sales Growth



Note: Figure B.6 shows different model generated moments of the sales growth and employment growth distribution. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a drop in the skewness, increase in variance, or both, in quarter 1, allowing normal evolution of the economy afterward. We plot the deviation of each macroeconomic aggregate from its value in quarter 0.

Appendix C

Appendix to Chapter 3

C.1 Worker Entry and Exit

Not all workers choose or are able to accept changes in wages at their current firm or a new firm in response to changes in firm productivity. Some workers respond – either willingly or not – by entering non-employment in the following period either by becoming unemployed or exiting labor force entirely. Clearly an analysis of wage changes does not really apply to this group of individuals. However, they are still exposed to the employment effects of firm-level TFP shocks, and this passthrough may be just as significant and heterogeneous as the passthrough to wages. So instead of estimating TFP changes effect on workers wage changes, we estimate if the probability of transitioning from employment to non-employment versus other firms becomes higher or lower when firms experience large TFP shocks. Furthermore, we investigate the effect of a firm's TFP changes on the probability of hiring workers out of non-employment relative to hiring from other firms. The regression model is as follows:

$$S_{ijt} = \alpha + \beta \Delta TFP_{jt} + X_{ijt}\gamma + \varepsilon_{ijt}$$

where S_{ijt} denotes the indicator of individual i's status change between period $t-1$ and t. For example, S_{ijt} is equal 1 if the worker switches from their current firm in period t into unemployment in $t+1$ and 0 otherwise. Alternately, it might indicate that a workers switched from unemployment in $t-1$ to their current firm in period t. The variable ΔTFP_{jt} indicates firm j's TFP change between $t-1$ and t whereas X_{ijt} includes the workers characteristics as well as the spousal characteristics we include in the selection model. The main parameter of interest is β, which measures the effect of firm level idiosyncratic TFP changes on worker probabilities of changing status.

The results of this analysis is shown in Table C.1. The top panel of Table C.1 shows the results for workers moving in and out of non-employment whereas the bottom panel shows the result for job switchers. The left two columns show the effect of TFP shocks on a worker's probability of moving into non-employment or another job. The sample here includes workers who will stay at their firm. The right two columns show the results for workers who move to firm j in period t. The sample here is workers who were not working at firm j in t-1. Looking at the top panel, the results suggest that the bigger the size of the TFP shock (positive or negative), the more likely workers will switch out of their firm into non-employment. Positive TFP shocks have a stronger effect on switching out: when firms experience a 1% TFP increase, the probability that workers move out to non-employment increases by 3.4%. A corresponding negative shocks also drives this up by 0.2%. This suggests that firms adjust their labor composition in reaction to large changes in TFP in either direction. The right two columns tell a strikingly different story where large changes in TFP make it more likely that a newly employed worker is coming from unemployment. Positive shocks increase the probability by 2.6% while 1% negative shocks also increase the probability by 1.5%. This is consistent with the churning story that when firms experiencing large TFP change, there will be more hiring and firing (churning). Looking at the bottom panel, we see that the TFP shocks have a very large effect on the probability of a worker leaving for another firm. As in the selection story, both large positive and negative shocks induce exit to other firms, which much larger magnitudes than for movements to non-employment. A 1% increase in TFP leads to a 19.3% increase in the probability that a worker switches to another firm. Finally, conditional on switching (as opposed to staying in the same firm between t-1 and t), workers entering a firm experiencing large positive shocks are less likely be coming from another firm relative to unemployment. However the are more likely to come from other firms if the TFP shock is negative. The asymmetry on the bottom panel is due to the differences in the sample in our analysis.

The results in this section further confirmed the importance of properly correcting for selection bias: when firms shocks are big (positive and negative), workers are more likely to switch out to another job and to unemployment. This biases our stayer and switcher analysis and therefore carefully correcting for this bias makes a stark difference in our results.

Table C.1: Moving In and Out of Employment

	(1)	(2)	(3)	(4)
	Move to		**Come from**	
	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$	$-\Delta TFP_{jt}$	$+\Delta TFP_{jt}$
	Non-employment			
β	-0.005^{***}	0.025^{***}	0.027^{***}	-0.011^{***}
	Other Job			
β	-0.102^{***}	0.152^{***}	0.004^{***}	0.034^{***}

Note: Table C.1 shows a set of linear probability regressions controlling for firm and worker characteristics. In columns 1-2, the dependent variable is an indicator which is 1 if the individual moves from employment to non-employment (top panel) or another employer (bottom panel). In columns 3-4, the dependent variable is an indicator which is 1 if the individual has moved to their current job from non-employment (top panel) or another employer (bottom panel). The main explanatory variable is the change in within-firm (log) TFP spanning the individual's transition into or out of the firm. $*p < 0.1, **p < 0.05, ***p < 0.01$. Robust standard errors are clustered at the firm level.

C.2 Model Extension

In this section, we consider the full model with selection. We allow for full asymmetric passthrough between positive and negative shocks, and we allow heterogeneous passthrough between stayers and switchers. This work is still ongoing.

Consider that log-wage of individual i that workers in firm j in period t is given by:

$$\begin{aligned}\log y_{i,j,t} =& d_t + X'_{i,t}\gamma + \eta_{i,t} + \varepsilon_{i,t} \\ &+\psi_{0i,j,s} z_{j,t-1} + \psi_{1i,j,s}\Delta z_{j,t}\mathbf{1}_{\Delta z_{j,t}>0}\mathbf{1}_{S_t} + \psi_{2i,j,s}\Delta z_{j,t}\mathbf{1}_{\Delta z_{j,t}\leq 0}\mathbf{1}_{S_t} \\ &+ \psi_{3i,j,s}\Delta z_{j,t}\mathbf{1}_{\Delta z_{j,t}>0}\mathbf{1}_{M_t} + \psi_{4i,j,s}\Delta z_{j,t}\mathbf{1}_{\Delta z_{j,t}\leq 0}\mathbf{1}_{M_t}\end{aligned} \tag{C.1}$$

d_t represents the average log price of human capital at time t, X_{it} is a set of workers' characteristics including age. The indicator $\mathbf{1}_{M_t}$ is equal to one when a worker is new to firm j. This wage process has a richer structure compare to the wage process in equation 3.13 in two ways: 1. it includes individual characteristics and time effects; 2. It allows the switchers and stayers to have different wage passthrough, in levels and changes. This implies:

$$
\begin{aligned}
\Delta \log y_{ijt} = {} & \Delta d_t + \Delta X_{it}'\gamma + \Delta\eta_{i,t} + \Delta\varepsilon_{it} \\
& + (\psi_1 \mathbf{1}_{\Delta z_{j,t}>0,S} + \psi_2 \mathbf{1}_{\Delta z_{j,t}\le 0,S} + \psi_3 \mathbf{1}_{\Delta z_{j,t}>0,M} + \psi_4 \mathbf{1}_{\Delta z_{j,t}\le 0,M})\Delta z_{j,t} \\
& + (\psi_0 - \psi_1 \mathbf{1}_{\Delta z_{j,t-1}>0,S} - \psi_2 \mathbf{1}_{\Delta z_{j,t-1}\le 0,S})\Delta z_{j,t-1} \\
& + (\psi_0 - \psi_1 \mathbf{1}_{\Delta z_{j,t-1}>0,M} - \psi_2 \mathbf{1}_{\Delta z_{j,t-1}\le 0,M})\Delta z_{j,t-1} \qquad \text{(C.2)}
\end{aligned}
$$

We could estimate all the parameters either using SMM (2 additional parameters, so add 2 more moments for movers), or use LMP's method, and directly back out all the parameters by algebra.